A solution to problems of non-participation in β^*-core mechanisms

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Abstract

To solve the free-rider problem, the literature has proposed various Lindahl mechanisms that attain Lindahl allocations for public good economies as equilibrium outcomes of the games. But Saijo and Yamato [1999] have proved that, for any Lindahl mechanism, some agents may have incentives not to participate in the mechanism.

To overcome such non-participation problems, Samejima [2004] has proposed to redesign Lindahl mechanisms. The present paper extends Samejima's result to general allocation problems, and shows that non-participation problems can be overcome for β^*-core mechanisms, that is, mechanisms that implement a sub-correspondence of the β^*-core. The β^*-core in the present paper is a subset of the β-core proposed by Aumann and Peleg [1960], but both cores are almost identical except for the treatment of indifference relations in the definition of blocking. An example of β^*-core mechanisms is a Lindahl mechanism proposed by Walker [1981]. The present paper shows how to redesign β^*-core mechanisms to overcome non-participation problems.

Our redesigned mechanisms have three significant properties. First, agents are given opportunities to show willingness not to participate in β^*-core mechanisms, possibly by passing over such willingness in silence. Second, 'non-participation decisions' are used as messages for the redesigned mechanisms. Third, the redesigned mechanisms respect the rights of non-participants in the sense that the mechanisms never interfere with actions of non-participants. In spite of these properties, the redesigned mechanisms can implement a sub-correspondence of the β^*-core in subgame-perfect equilibrium.

1. Introduction

This paper proposes a solution to non-participation problems for mechanisms that attain β^*-core allocations in a general allocation model.

Lindahl equilibrium for public good economies has desirable properties such as Pareto efficiency and individual rationality. However, in Lindahl equilibria, agents have incentives to under-report their valuations of public goods. Such incentives to misrepresent their preferences result in under-provisions of the public goods. This is called the free-rider problem.

To solve the free-rider problem, economists have designed various *mechanisms* that attain Lindahl allocations as equilibrium outcomes of the games. A mechanism in the field of game theory is also called a *game form.* It specifies rules of the game; A game form consists of *message spaces*[1] and an *outcome function.*[2] Lindahl mechanisms, which attain Lindahl allocations as equilibrium outcomes of the games, have been proposed by Hurwicz [1979], Walker [1981], and so on.

However, Saijo and Yamato [1999] have proved that, for any Lindahl mechanism, some agents may have incentives not to participate in the mechanism. The previous studies on mechanism design implicitly assume that all agents participate in the proposed mechanism. If agents have the freedom of non-participation, then they might not participate, knowing that they can free-ride on the public goods provided by participants. This is what Saijo and Yamato call *non-participation problems.*

Why is non-participation a serious problem? We would like to point out two reasons. The first reason is *no-message problem.* Since the mechanisms may not be able to get adequate messages from non-participants, the mechanisms may not select a Lindahl allocation for the society as an outcome of a message profile of all agents. The second reason is *no-tax problem.* Since the mechanisms cannot tax non-participants and use their private resources for the provision of public goods, the mechanisms may end up allowing non-participants to free-ride on the public goods provided by participants.

To overcome such non-participation problems, Samejima [2004] has proposed to redesign Lindahl mechanisms. The redesigned mechanism incorporates the existing Lindahl mechanism as a component, and offers a solution to problems of non-participation.

As a solution to no-message problem, Samejima has proposed to use *'participation/non-participation decisions'* as messages for the redesigned mechanisms. By doing so, the redesigned mechanisms can receive *some* messages from all agents including both participants and non-participants in the incorporated Lindahl mechanisms. Although every agent may be able to choose to reveal no message for the incorporated Lindahl mechanism, every agent necessarily reveals some message (i.e. participation/non-participation decision) for the redesigned mechanism.

As for no-tax problem, there is not a solution because controlling private resources of *non-participants* is considered to be an infringement of their property rights. But it is reasonable to assume that mechanisms are allowed to control private resources of *participants* because they have chosen participation

[1]Message spaces specify what kind of messages each agent should submit to the mechanism as his strategy.
[2]An outcome function in the standard studies specifies which allocation the mechanism selects as an outcome of a given message profile.

voluntarily with an agreement to follow the rules of the mechanisms. Such an assumption is standard in the literature. Based on this assumption, Samejima's redesigned mechanisms control private resources of agents who have explicitly chosen to participate in the incorporated Lindahl mechanisms while the redesigned mechanisms do not control private resources of agents who have chosen not to participate in the incorporated Lindahl mechanisms. In the redesigned mechanisms, it is possible to control participants' resources so that non-participants are induced to change their decisions and choose participation in the incorporated Lindahl mechanisms.

The present paper extends the result by Samejima [2004] from public good provision problems to general allocation problems. We show that for any mechanism that attains β^*-core allocations as equilibrium outcomes, we can redesign the mechanism so that non-participation problems do not occur. The β^*-core is almost identical to the β-core proposed by Aumann and Peleg [1960] for cooperative games without side payments. The difference between the β^*-core and the β-core lies in the treatment of indifference relations in the definition of blocking.

In addition to the generality of the model, the present paper differs from Samejima [2004] in that we use equilibrium notions that are more standard in the literature. Samejima [2004] has defined *order-independent subgame-perfect equilibrium* for his analysis, but the present paper employs *subgame-perfect equilibrium* proposed by Selten [1975]. Furthermore, in its attempt to generalize the model, Samejima [2004] has defined the *core under a*, where a represents a specific joint action profile, but this notion of the core is far from standard. The present paper employs the β^*-core, which is almost identical to the β-core that is classical in the literature.

The remaining part of this paper is organized as follows. Section 2 explains a model of general allocation problems and introduces the notions of the β^*-core and mechanisms. Section 3 presents our result, and Section 4 concludes.

2. The Model

2–1. The general environment

The set of *agents* in the society is denoted by $N = \{1, 2, \dots, n\}$. We assume that $|N| \geq 2$. A *coalition* is a non-empty subset of N and it is denoted typically by S. The set of *allocations* in the society is denoted by $\bar{X}$. For example, an allocation $x \in \bar{X}$ represents a distribution of private goods among agents and an amount of public goods provided in the society.

We assume that an allocation is realized as a result of actions of *all* agents. The set of *feasible actions* for a coalition S is denoted by A_S. For notational convenience, $a_{\{i\}}$ and $A_{\{i\}}$ are simply denoted by a_i

and A_i, respectively. For example, agent i's action $a_i \in A_i$ represents his decision about an amount of private consumption of his endowments and an amount of his contributions for the provision of public goods in the society. Let $h\colon A_N \to \bar{X}$ be a function that assigns an allocation to each action of N. We assume that an action of a proper subset $S \subsetneq N$ of agents cannot determine a specific allocation in the society. That is, it is actions of *all* agents that can determine an allocation in the society. This is because an allocation may represent an amount of goods with externalities: non-excludable public goods, air pollution, and so on. For example, an amount of pollution in the society is determined by summing up an amount of pollution emitted by all agents, not by a proper subset of agents. We say that an allocation $x \in \bar{X}$ is *feasible* if $x = h(a_N)$ for some $a_N \in A_N$. The set of *feasible allocations* is denoted by X.

Each agent i has a complete and transitive *preference relation* R_i defined over $\bar{X}$. As usual, $x\, R_i\, x'$ means that x is at least as good as x' for agent i. The associated strict preference relation is denoted by P_i. That is, $x\, P_i\, x'$ means that agent i prefers x to x'. Let $\mathcal{R}_i$ be the set of *admissible preferences* of agent i. A *preference profile* of a coalition S is denoted by $R_S \equiv (R_i)_{i \in S}$ and the set of admissible preference profiles of a coalition S is denoted by $\mathcal{R}_S \equiv \times_{i \in S} \mathcal{R}_i$.

A *social choice correspondence* $\varphi\colon \mathcal{R}_N \rightrightarrows X$ is a set-valued function that assigns a non-empty subset of feasible allocations to each profile of preferences of all agents.

A coalition S is said to *β-block a feasible allocation* $x \in X$ if for each $a_{N\setminus S} \in A_{N\setminus S}$, there exists $a_S \in A_S$ such that $h(a_S, a_{N\setminus S})\, P_i\, x$ for all $i \in S$.[3] A feasible allocation $x \in X$ is a *β-core allocation* if no coalition can β-block x. Given a preference profile $R_N \in \mathcal{R}_N$, we obtain the set of β-core allocations, which we denote by $C(R_N)$.

The above definition of β-blocking is standard in the literature, but we consider a slightly different definition to obtain the result of the present paper. A coalition S is said to *β^*-block a feasible allocation* $x \in X$ if for each $a_{N\setminus S} \in A_{N\setminus S}$, there exists $a_S \in A_S$ such that $h(a_S, a_{N\setminus S}) \neq x$ and $h(a_S, a_{N\setminus S})\, R_i\, x$ for all $i \in S$. A feasible allocation $x \in X$ is a *β^*-core allocation* if no coalition can β^*-block x. Given a preference profile $R_N \in \mathcal{R}_N$, we obtain the set of β^*-core allocations, which we denote by $C^*(R_N)$. The difference between the β^*-core and the β-core lies in the treatment of indifference relations in the definition of blocking. Note that if S can β-block x, then S can also β^*-block x. Therefore, $C^*(R_N) \subset C(R_N)$.

In general, the set of β^*-core allocations may be the empty set.[4] But if the set of β^*-core alloca-

[3] In the definition of β-blocking as well as in the definition of β^*-blocking to follow, we allow the case $S = N$. In this case, the phrase 'for each $a_{N\setminus S} \in A_{N\setminus S}$' is vacuous.

[4] In some models, the existence of β^*-core allocations is guaranteed as we discuss in Section 2–2.

tions is non-empty and a social choice correspondence φ selects β^*-core allocations for all admissible preference profiles, that is, if $\varphi(R_N) \subset C^*(R_N) \neq \emptyset$ for all $R_N \in \mathcal{R}_N$, then we say that φ is a *sub-correspondence of the β^*-core.*

A *mechanism* Γ is an extensive *game form* as studied in Moore and Repullo [1988]. The mechanism consists of a game tree with a set of message choices available to agents at each information set, an outcome function, and a set of participants at the initial node. Unlike the standard studies in the literature, we consider the possibility that the set of participants in the mechanism may vary as play proceeds.

A *participant* in the mechanism is defined as an agent who has voluntarily and explicitly chosen to follow the rules of the mechanism. In other words, a participant i is an agent who has allowed the mechanism to control his action. An agent who is not a participant is a *non-participant.* For each node t in the game tree, we denote the set of *participants at node* t by $T(t)$. Particularly, the *set of participants at the initial node of the mechanism* is denoted by T. If a non-participant has an opportunity in the game tree to show his willingness to participate, then he can become a participant. So, if a node t is a predecessor of a node t', then we have $T(t) \subset T(t')$.[5] The standard studies assume that all agents are participants at the initial node, that is, $T = N$. However, in Section 3, we will consider the case where no agent is a participant at the initial node, that is, $T = \emptyset$.

The *outcome function* of the mechanism associates with each terminal node t an action of the set of participants, $a^t_{T(t)} \in A_{T(t)}$. So, at the terminal node t, the mechanism controls actions of the participants in $T(t)$ while it never interferes with actions of the non-participants in $N \setminus T(t)$. We assume that the set of non-participants will take some action $a^t_{N\setminus T(t)} \in A_{N\setminus T(t)}$ for their own benefit after watching the action $a^t_{T(t)}$ of the participants.[6]

Agent i's *strategy* m_i is a function that associates agent i's message choice from available choices with each information set for which he is on the move. We do not consider mixed strategies in the present paper. The set of agent i's strategies is also called agent i's *message space* and denoted by M_i. We note that in the above definition of agent i's strategy, we do not assume that he is a participant. This is because in the redesigned mechanism that we discuss in Section 3, a non-participant inevitably chooses a strategy, where he reveals a message telling that he has chosen non-participation.

[5]In other words, non-participants can become participants but not vice versa. We consider that this assumption may not be unreasonable because participants have voluntarily allowed the mechanism to control their actions. Furthermore, we note that the assumption in the standard studies is much more stronger; They take it for granted that all agents are participants at the initial node.

[6]To obtain our result in Section 3, it does not matter whether the non-participants choose $a^t_{N\setminus T(t)}$ cooperatively or non-cooperatively.

A *strategy profile* of a coalition S of agents is denoted by $m_S \equiv (m_i)_{i\in S}$. The set of strategy profiles of S is denoted by $M_S \equiv \times_{i\in S} M_i$, which is also called *message spaces of S*. Given a strategy profile m_N of all agents, we can associate with m_N a terminal node t, and by using the outcome function, we can also associate with m_N an action $a^t_{T(t)}$ of the participants at the terminal node t. Furthermore, we can associate with m_N an action $a^t_{N\setminus T(t)}$ that the non-participants at the terminal node t choose for their own benefit after watching $a^t_{T(t)}$. So, we can define an *outcome allocation function* $g\colon M_N \to X$ as follows; For each $m_N \in M_N$, let $g(m_N) \equiv h(a^t_{T(t)}, a^t_{N\setminus T(t)})$, where $T(t)$ is the set of participants at the terminal node t associated with m_N.

A list (Γ, R_N) defines an extensive *game*. A strategy profile m_N is a *Nash equilibrium* of (Γ, R_N) if no agent can benefit by changing his strategy while the others keep their strategies unchanged; That is, $g(m_N)\ R_i\ g(m'_i, m_{N\setminus\{i\}})$ for all $i \in N$ and for all $m'_i \in M_i$. A strategy profile m_N is a *subgame-perfect equilibrium* of (Γ, R_N) if in every subgame of (Γ, R_N), the strategy profile induced by m_N is a Nash equilibrium. By definition, a strategy profile m_N is a subgame-perfect equilibrium of (Γ, R_N) if and only if the strategy profile induced by m_N is a subgame-perfect equilibrium in every subgame of (Γ, R_N). Let $NE(\Gamma, R_N)$ be the set of *Nash equilibrium allocations* corresponding to the Nash equilibria of the game (Γ, R_N).[7] Let $SPE(\Gamma, R_N)$ be the set of *subgame-perfect equilibrium allocations* corresponding to the subgame-perfect equilibria of the game (Γ, R_N).[8]

We say that a mechanism Γ *implements* a social choice correspondence φ *in Nash equilibrium* if $NE(\Gamma, R_N) = \varphi(R_N)$ for all $R_N \in \mathcal{R}_N$. Similarly, we say that a mechanism Γ *implements* a social choice correspondence φ *in subgame-perfect equilibrium* if $SPE(\Gamma, R_N) = \varphi(R_N)$ for all $R_N \in \mathcal{R}_N$. These notions of implementation are called *full implementation* in the literature.

2–2. An example of the β^*-core in public good economies

This section shows that Lindahl allocations are β^*-core allocations in public good economies with strictly convex preferences defined over consumption spaces with one private good and one pure public good.

Each agent $i \in N$ consumes one private good $x_i \in \mathbb{R}_+$ and one pure public good $y \in \mathbb{R}_+$, where y should be common among all agents.[9] An allocation in the society is denoted typically by $x \equiv ((x_i)_{i\in N}, y)$ and the set of allocations is denoted by $\bar{X} = \mathbb{R}^{n+1}_+$. Each agent i is initially endowed with a positive amount $w_i > 0$ of the private good and no public good, $y = 0$. Define $w \equiv \sum_{i\in N} w_i$.

[7]That is, $NE(\Gamma, R_N) \equiv \{x \in X : x = g(m_N)$ for some Nash equilibrium m_N of $(\Gamma, R_N)\}$.
[8]That is, $SPE(\Gamma, R_N) \equiv \{x \in X : x = g(m_N)$ for some subgame-perfect equilibrium m_N of $(\Gamma, R_N)\}$.
[9]$\mathbb{R}_+$ denotes the set of non-negative real numbers.

Each agent i's preference relation R_i defined over $\bar{X}$ is represented by a utility function $u_i(x_i, y)$ where u_i is increasing in both arguments and u_i is strictly quasi-concave.[10] That is, agent i's preference relation R_i is strongly monotone and strictly convex over his consumption space $\mathbb{R}^2_+$.

Agent i's feasible action is $a_i = (x_i, c_i) \in A_i \equiv \{(x_i, c_i) \in \mathbb{R}^2_+ : x_i + c_i \leq w_i\}$ where c_i is the amount of his private good that he contributes for the production of the public good.[11] Define $c \equiv \sum_{i \in N} c_i$. An action of a coalition S is denoted by $a_S = (a_i)_{i \in S} \in A_S \equiv \times_{i \in S} A_i$.

There is a production technology that transforms the private good into the public good, which is represented by an increasing, concave production function $f\colon \mathbb{R}_+ \to \mathbb{R}_+$ with $f(0) = 0$. As a result of an action $a_N \in A_N$ of all agents, they realize an allocation $h(a_N) = ((x_i)_{i \in N}, y) \in \bar{X}$ where $y = f(c)$. The set of feasible allocations is denoted by $X \equiv \{x \in \bar{X} : x = h(a_N) \text{ for some } a_N \in A_N\}$.

We now define a Lindahl equilibrium. Let $p_0 \in \mathbb{R}_+$ denote a price of the private good and let $p_i \in \mathbb{R}_+$ denote a *personalized price* of the public good for agent $i \in N$. A *Lindahl equilibrium* with respect to $(w_i)_{i \in N}$ is a feasible allocation $x = ((x_i)_{i \in N}, y)$ and a price system $(p_0, (p_i)_{i \in N})$ with the following properties, L1, L2, and L3.

L1. *Profit maximization.* $y = f(c)$ and $\sum_{i \in N} p_i \cdot y - p_0 \cdot c \geq \sum_{i \in N} p_i \cdot y' - p_0 \cdot c'$ for all (y', c') with $y' = f(c')$ and $c' \in [0, w]$.

L2. *Utility maximization.* For all $i \in N$, we have $p_0 \cdot x_i + p_i \cdot y \leq p_0 \cdot w_i$, and if $u_i(x'_i, y') > u_i(x_i, y)$, then $p_0 \cdot x'_i + p_i \cdot y' > p_0 \cdot w_i$.

L3. *Market clearing.* $\sum_{i \in N} x_i + c = w$.

A *Lindahl allocation* is a feasible allocation $x = ((x_i)_{i \in N}, y)$ for which there exists a price system $(p_0, (p_i)_{i \in N})$ that satisfies L1, L2, and L3. The existence of a Lindahl equilibrium in the current setting can be shown by applying the theorem due to Foley [1970] with some modifications. The following proposition is also a modification of Foley's result. However, we would like to emphasize the importance of strict convexity of preferences to obtain the proposition.

Proposition 1. *A Lindahl allocation is a β^*-core allocation.*

Proof. Let $x = ((x_i)_{i \in N}, y)$ and $(p_0, (p_i)_{i \in N})$ be a Lindahl equilibrium. By way of contradiction, suppose that there exists a coalition $S \subset N$, possibly $S = N$, that can β^*-block x.

Let $a_{N \setminus S} = ((x'_i, c'_i))_{i \in N \setminus S}$ be such that $(x'_i, c'_i) = (w_i, 0)$ for all $i \in N \setminus S$. Since S can β^*-block x, there exists $a_S = ((x'_i, c'_i))_{i \in S} \in A_S$ such that $h(a_S, a_{N \setminus S}) \neq x$ and $h(a_S, a_{N \setminus S})\ R_i\ x$ for all $i \in S$.

[10] We say that u_i is *strictly quasi-concave* if $u_i(\alpha x_i + (1-\alpha)x'_i, \alpha y + (1-\alpha)y') > \min\{u_i(x_i, y), u_i(x'_i, y')\}$ for all $\alpha \in (0,1)$ and for all (x_i, y) and (x'_i, y') such that $(x_i, y) \neq (x'_i, y')$.

[11] In our definition of A_i, free disposal of his private good is admitted.

Define $c' = \sum_{i\in N} c'_i$ and $y' = f(c')$. Note that $h(a_S, a_{N\setminus S}) = ((x'_i)_{i\in N}, y')$. So, we must have the fact that $((x'_i)_{i\in N}, y') \neq ((x_i)_{i\in N}, y)$ and $u_i(x'_i, y') \geq u_i(x_i, y)$ for all $i \in S$.

We first claim that $\sum_{i\in N} p_i \cdot y' + p_0 \cdot \sum_{i\in S}(x'_i - w_i) \leq 0$. By L2 and strong monotonicity of preferences, we have $p_0 \cdot x_i + p_i \cdot y = p_0 \cdot w_i$ for all $i \in N$ and hence $\sum_{i\in N} p_i \cdot y + p_0 \cdot \sum_{i\in N}(x_i - w_i) = 0$. This equality and L3 imply that $\sum_{i\in N} p_i \cdot y - p_0 \cdot c = 0$. Furthermore, by L1 and the fact that $x'_i + c'_i \leq w_i$ for all $i \in N$, we obtain $0 \geq \sum_{i\in N} p_i \cdot y' - p_0 \cdot c' \geq \sum_{i\in N} p_i \cdot y' + p_0 \cdot \sum_{i\in N}(x'_i - w_i)$. Since $x'_i = w_i$ for all $i \in N \setminus S$, we obtain the claim.

We next claim that $\sum_{i\in N} p_i \cdot y' + p_0 \cdot \sum_{i\in S}(x'_i - w_i) \geq 0$. By L2 and strong monotonicity of preferences, we have $p_0 \cdot x'_i + p_i \cdot y' \geq p_0 \cdot w_i$ for all $i \in S$ and hence $\sum_{i\in S} p_i \cdot y' + p_0 \cdot \sum_{i\in S}(x'_i - w_i) \geq 0$. Since $p_i \cdot y' \geq 0$ for all $i \in N \setminus S$, we obtain the claim.

The previous two claims imply that $\sum_{i\in N} p_i \cdot y' + p_0 \cdot \sum_{i\in S}(x'_i - w_i) = 0$. Furthermore, since $p_0 \cdot x'_i + p_i \cdot y' \geq p_0 \cdot w_i$ for all $i \in S$ and $p_i \cdot y' \geq 0$ for all $i \in N \setminus S$ as we noted previously, it must be the case that $p_0 \cdot x'_i + p_i \cdot y' = p_0 \cdot w_i$ for all $i \in S$ and $p_i \cdot y' = 0$ for all $i \in N \setminus S$.

We claim that $(x'_i, y') = (x_i, y)$ for all $i \in S$. Suppose that there is $j \in S$ such that $(x'_j, y') \neq (x_j, y)$. Let $(x''_j, y'') = (0.5x'_j + 0.5x_j, 0.5y' + 0.5y)$. By strict convexity of agent j's preference, that is, by strict quasi-concavity of u_j, we have the fact that $u_j(x''_j, y'') > \min\{u_j(x'_j, y'), u_j(x_j, y)\} = u_j(x_j, y)$, which in turn implies that $p_0 \cdot x''_j + p_j \cdot y'' > p_0 \cdot w_j$ by L2. However, this inequality is in contradiction with the equality $p_0 \cdot x''_j + p_j \cdot y'' = p_0 \cdot w_j$, which must hold since $p_0 \cdot x'_j + p_j \cdot y' = p_0 \cdot w_j$ and $p_0 \cdot x_j + p_j \cdot y = p_0 \cdot w_j$.

By the last claim, we must have $y' = y$. We next show that $y' = y = 0$. If $y' > 0$, then, for all $i \in N \setminus S$, $p_i \cdot y' = 0$ implies $p_i = 0$. When $p_i = 0$, there does not exist (x_i, y) that satisfies L2 since agent i's preference is strongly monotone. This non-existence is in contradiction with our assumption that x is a Lindahl allocation. So, we must have $y' = y = 0$.

The fact that $p_0 \cdot x_i + p_i \cdot y = p_0 \cdot w_i$ together with $y = 0$ implies that $x_i = w_i$ for all $i \in N$. Recall that we have $x'_i = w_i$ for all $i \in N \setminus S$ and $(x'_i, y') = (x_i, y)$ for all $i \in S$. So, we obtain $((x'_i)_{i\in N}, y') = ((x_i)_{i\in N}, y)$. However, this contradicts the fact that $h(a_S, a_{N\setminus S}) \neq x$. □

3. The Result

3–1. The mechanism

Throughout the remaining part of this paper, let a social choice correspondence φ be a compact-valued[12] sub-correspondence of the β^*-core, and let a mechanism Γ be a game form that implements

[12] We say that φ is *compact-valued* if $\varphi(R_N)$ is a compact set for all $R_N \in \mathcal{R}_N$.

φ in subgame-perfect equilibrium. That is, $SPE(\Gamma, R_N) = \varphi(R_N) \subset C^*(R_N) \neq \emptyset$ for all $R_N \in \mathcal{R}_N$. We allow that the mechanism Γ to be a single-stage simultaneous-move game form that implements φ in Nash equilibrium because, for such a game form, we have the fact that $NE(\Gamma, R_N) = SPE(\Gamma, R_N)$ for all $R_N \in \mathcal{R}_N$. We assume that the mechanism Γ consists of the set of strategy profiles of all agents M_N, the associated outcome allocation function $g\colon M_N \to X$, and the set of participants at the initial node $T = N$. We call the mechanism Γ a *β^*-core mechanism.* Examples of β^*-core mechanisms include Lindahl mechanisms proposed by Hurwicz [1979] and Walker [1981] in public good economies with strictly convex preferences discussed in Section 2–2. Since $T = N$ is assumed for Γ, it is assumed that all agents are participants at the initial node of the mechanism as in the standard studies.

We introduce some notations. For each $R_N \in \mathcal{R}_N$, we define the *least preferred allocation in $\varphi(R_N)$ for agent i*, which is denoted by $\underline{x}_i(R_N)$.[13] That is, we have $\underline{x}_i(R_N) \in \varphi(R_N)$ and $x\ R_i\ \underline{x}_i(R_N)$ for all $x \in \varphi(R_N)$.

For each $R_N \in \mathcal{R}_N$, $x \in C^*(R_N)$, and a coalition $S \subsetneq N$, we denote by $\underline{a}_{N\setminus S}(R_N, x)$ an *action of $N \setminus S$ that prevents S from β^*-blocking x under R_N.* That is, for all $a_S \in A_S$, we have either $x = h(a_S, \underline{a}_{N\setminus S}(R_N, x))$ or $x\ P_i\ h(a_S, \underline{a}_{N\setminus S}(R_N, x))$ for some $i \in S$. The existence of $\underline{a}_{N\setminus S}(R_N, x)$ is guaranteed because $x \in C^*(R_N)$ and hence S cannot β^*-block x.[14]

We now describe how to redesign the β^*-core mechanism Γ and obtain Γ^*. We emphasize three significant properties of the redesigned mechanism Γ^*.

Property 1. $T = \emptyset$ is assumed for Γ^*, so no agent is a participant at the initial node of Γ^*. Each agent is given an opportunity in Γ^* to show his willingness to become a participant.

Property 2. Since 'participation/non-participation decisions' are used as messages for Γ^*, every agent inevitably reveals some message. Even if some agent tries to reveal no message by remaining silent, such a behavior is interpreted as choosing non-participation. So, no-message problem does not occur.

Property 3. At each terminal node t, the redesigned mechanism controls an action $a^t_{T(t)}$ of the participants at the node t. Since Γ^* never interferes with actions of the non-participants, and since the non-participants can freely choose their action $a^t_{N\setminus T(t)}$ for their own benefit, we could say that the rights of the non-participants are respected. Furthermore, since each participant has voluntarily chosen participation, his own free will is respected, too.

[13]If there are multiple least preferred allocations in $\varphi(R_N)$ for agent i, we pick one of them arbitrarily and denote it by $\underline{x}_i(R_N)$.

[14]If there are multiple actions of $N \setminus S$ that prevent S from β^*-blocking x under R_N, we pick one of them arbitrarily and denote it by $\underline{a}_{N\setminus S}(R_N, x)$.

The redesigned mechanism Γ^* based on Γ.

The redesigned mechanism Γ^* consists a game tree with message spaces M_N^*, an outcome function, and the empty set of participants at the initial node. That is, no agent is regarded as a participant at the beginning. The following descriptions define the game tree, M_N^*, and the outcome function.

Phase 1. This is a simultaneous-move phase. Each agent i is asked to report $(z^i, R_N^i, x^i) \in \mathbb{N}^0 \times \mathcal{R}_N \times X$ simultaneously, where $\mathbb{N}^0$ denotes the set of non-negative integers and it is required that $x^i \in \varphi(R_N^i)$. If $z^i = 0$, agent i is regarded as choosing to become a participant. If $z^i \geq 1$, agent i is regarded as choosing to become a non-participant. Each agent i has the right to remain silent completely or partially, but then he is regarded as choosing $z^i = 1$ to become a non-participant, and (R_N^i, x^i) with $x^i \in \varphi(R_N^i)$ is chosen by the mechanism randomly. If agent i's report does not conform to the rules, for example, if $x^i \notin \varphi(R_N^i)$, then he is also regarded as choosing to become a non-participant and his message is replaced by a randomly chosen message (z^i, R_N^i, x^i) with $z^i = 1$ and $x^i \in \varphi(R_N^i)$.

Case 1. If $z^i = 0$ for all $i \in N$, then the play proceeds to phase 2–1. This is the case where all agents have chosen participation.

Case 2. If there exists $j \in N$ such that $z^i = 0 \neq z^j$ for all $i \in N \setminus \{j\}$, then the play stops. This is the case where only agent j has chosen non-participation.

At this terminal node t, the outcome function defines an action of the set of participants $T(t) = N \setminus \{j\}$ by $a_{T(t)}^t = \underline{a}_{N\setminus\{j\}}(R_N^{j-1}, \underline{x}_j(R_N^{j-1}))$ where R_N^{j-1} is interpreted as R_N^n if $j = 1$.[15] In words, the outcome at this terminal node is such that participants take actions that prevent agent j from β^*-blocking his least preferred allocation in $\varphi(R_N^{j-1})$ where R_N^{j-1} is a preference profile reported by another agent next to j.

Case 3. If neither case 1 nor 2 applies, then the play proceeds to phase 2–2. This is the case where at least two agents have chosen non-participation.

Phase 2–1. This phase is reached when case 1 applies in phase 1. So, all agents are participants at the beginning of this phase. Here, the β^*-core mechanism Γ is used as a component of the redesigned mechanism Γ^*. All agents are asked to play a game in Γ with message spaces M_N and an outcome allocation function g. When the game in Γ reaches its terminal node, the play in Γ^* also stops.

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[15] We note that $\underline{a}_{N\setminus\{j\}}(R_N^{j-1}, \underline{x}_j(R_N^{j-1}))$ is well-defined since $\underline{x}_j(R_N^{j-1}) \in \varphi(R_N^{j-1}) \subset C^*(R_N^{j-1})$.

At each terminal node in Γ, the outcome function of Γ^* defines actions of all agents so that the resulting outcome allocation conforms to g.

Phase 2–2. This phase is reached when case 3 applies in phase 1. So, at least two agents are non-participants at the beginning of this phase. Denote the set and the number of non-participants at the beginning of this phase by $S_1 \equiv \{i \in N \colon z^i \geq 1\}$ and $\bar{k} \equiv |S_1|$, respectively. Let agent i^* be the non-participant with the least index number among those who have reported the largest integer in phase 1. That is, $i^* \equiv \min\{i \in S_1 \colon i \in \arg\max_{j \in S_1} z^j\}$. Phase 2–2 consists of $\bar{k}$ rounds described as follows.

Round $k \in \{1, 2, \ldots, \bar{k} - 1\}$. Each non-participant $i \in S_k$ is asked to report $z^i_k \in \{0, 1\}$ sequentially from the non-participant with the least index number to those with larger ones.

If $z^i_k = 0$, non-participant i is regarded as changing his mind and choosing to become a participant. In this case, the set of non-participants is updated, i.e., $S_{k+1} \equiv S_k \setminus \{i\}$, and the play proceeds to the next round $k+1$ even if some non-participants are not yet asked in the ongoing round k.

If $z^i_k = 1$, non-participant i is regarded as choosing to remain a non-participant. If non-participant i remains silent or his report does not conform to the rules, then he is regarded as choosing $z^i_k = 1$ to remain a non-participant. In this case, another non-participant $j \in S_k$ with a next larger index number is asked to report $z^j_k \in \{0, 1\}$ in the ongoing round k.

Non-participant j is treated in the same way as non-participant i. If $z^j_k = 0$, he becomes a participant and the play proceeds to the next round $k+1$ with the updated set of non-participants $S_{k+1} \equiv S_k \setminus \{j\}$. If $z^j_k = 1$, another non-participant is asked in the ongoing round k. The play in the round k continues similarly, either until the next round $k+1$ is reached after some non-participant chooses participation, or until all non-participants in S_k sequentially choose non-participation in a row.

The play stops in the round k without proceeding to the next round only when all non-participants in S_k sequentially choose non-participation in a row, that is, only when $z^i_k = 1$ for all $i \in S_k$. At this terminal node t, the outcome function defines an action of the set of participants $T(t) = N \setminus S_k$ by $a^t_{T(t)} = \underline{a}_{N \setminus S_k}(R^{i^*}_N, x^{i^*})$.[16] In words, the outcome at this terminal node is such that participants take actions that prevent a coalition of non-participants S_k from β^*-blocking x^{i^*} under $R^{i^*}_N$. In the special case where $T(t) = \emptyset$ at the terminal node t, the outcome function associates nothing with the node t since no agent is a participant at t. Such a case occurs only when $S_1 = N$ and the play stops in round 1.

[16] We note that $\underline{a}_{N \setminus S_k}(R^{i^*}_N, x^{i^*})$ is well-defined since $x^{i^*} \in \varphi(R^{i^*}_N) \subset C^*(R^{i^*}_N)$.

Round $\bar{k}$. In this round, there remains only one non-participant i in $S_{\bar{k}}$. He is asked to report $z^i_{\bar{k}} \in \{0, 1\}$.

If $z^i_{\bar{k}} = 0$, non-participant i becomes the last participant and the play stops. In this case, all agents have chosen participation in the end. At this terminal node t, the outcome function defines an action $a^t_{T(t)}$ of the set of participants $T(t) = N$ so that $h(a^t_{T(t)}) = x^{i^*}$ and the outcome allocation x^{i^*} is realized.

If $z^i_{\bar{k}} = 1$, non-participant i remains a non-participant and the play stops. At this terminal node t, the outcome function defines an action of the set of participants $T(t) = N \setminus \{i\}$ by $a^t_{T(t)} = \underline{a}_{N\setminus\{i\}}(R^{i^*}_N, x^{i^*})$.

Theorem. *Suppose that φ be a compact-valued sub-correspondence of the β^*-core. Let Γ^* be the redesigned mechanism obtained from a β^*-core mechanism Γ, for which we have the fact that $SPE(\Gamma, R_N) = \varphi(R_N) \subset C^*(R_N) \neq \emptyset$ for all $R_N \in \mathcal{R}_N$. Then, the redesigned mechanism Γ^*, which has Properties 1, 2 and 3, implements φ in subgame-perfect equilibrium. That is, $SPE(\Gamma^*, R_N) = \varphi(R_N)$ for all $R_N \in \mathcal{R}_N$.*

This theorem is our main result. The following two sections prove the theorem.

3–2. Proof: $SPE(\Gamma^*, R_N) \subset \varphi(R_N)$ for all $R_N \in \mathcal{R}_N$.

Choose any arbitrary $R_N \in \mathcal{R}_N$ and fix it throughout this section. This R_N is regarded as a 'true' preference profile of agents.

We first focus on subgames that start at the beginning of round 1 in phase 2–2. We note that there are infinitely many such subgames because case 3 applies for infinitely many message profiles reported in phase 1. Before any such subgame starts, the set of non-participants S_1, the number of non-participants $\bar{k}$, and non-participant i^* who has reported the largest integer are already given.

Lemma 1. *For any subgame that starts at the beginning of round 1 in phase 2–2, if $R^{i^*}_N = R_N$ and $x^{i^*} \in \varphi(R_N)$, then the outcome allocation is x^{i^*} in the subgame-perfect equilibria of this subgame given that the 'true' preference profile of agents is R_N.*

Proof. We use backward induction arguments to prove the lemma.

First, consider any subgame that starts at the beginning of round $\bar{k}$ and take any subgame-perfect equilibrium for this subgame. In round $\bar{k}$, there remains only one non-participant i, who is asked to report $z^i_{\bar{k}} \in \{0, 1\}$.

If $z^i_{\bar{k}} = 0$ on the equilibrium path, then the outcome allocation is x^{i^*} as desired.

If $z^i_{\bar{k}} = 1$ on the equilibrium path, then the outcome allocation is $x' \equiv h(a^t_i, \underline{a}_{N\setminus\{i\}}(R^{i^*}_N, x^{i^*}))$ where a^t_i is chosen by non-participant i for his own benefit at the terminal node t. Since non-participant i has an opportunity to change his report into $z^i_{\bar{k}} = 0$ and realize x^{i^*} but the equilibrium allocation is x', it must be the case that non-participant i weakly prefers x' to x^{i^*}, that is, $x' \, R_i \, x^{i^*}$. By the definition of $\underline{a}_{N\setminus\{i\}}(R^{i^*}_N, x^{i^*})$ and by the assumption $R^{i^*}_N = R_N$ of the lemma, we must have the fact that $x' = x^{i^*}$ as desired, because it is not the case that $x^{i^*} \, P_i \, x'$.

Second, assume that it has been proved that the subgame-perfect equilibrium allocation is x^{i^*} for every subgame that starts at the beginning of round $k+1$, where k is some number in $\{1, \dots, \bar{k}-1\}$. Consider any subgame that starts at the beginning of round k and take any subgame-perfect equilibrium for this subgame. In round k, each non-participant $i \in S_k$ is asked to report $z^i_k \in \{0, 1\}$ sequentially.

If $z^i_k = 0$ for some $i \in S_k$ on the equilibrium path, that is, if any one non-participant chooses to become a participant in round k on the equilibrium path, then round $k+1$ is reached on the path. By the induction assumption, the outcome allocation is x^{i^*} as desired.

If $z^i_k = 1$ for all $i \in S_k$ on the equilibrium path, that is, if all non-participants in S_k sequentially choose non-participation in a row in round k on the equilibrium path, then the outcome allocation is $x' \equiv h(a^t_{S_k}, \underline{a}_{N\setminus S_k}(R^{i^*}_N, x^{i^*}))$ where $a^t_{S_k}$ is chosen by the non-participants in S_k for their own benefit at the terminal node t. Note that each non-participant $i \in S_k$ has an opportunity to change his report into $z^i_k = 0$ and reach round $k+1$ to realize x^{i^*}, which is, by the induction assumption, the subgame-perfect equilibrium allocation of every subgame that starts at the beginning of round $k+1$. Nevertheless, the equilibrium allocation is not x^{i^*} but x'. So, it must be the case that every non-participant $i \in S_k$ weakly prefers x' to x^{i^*}, that is, $x' \, R_i \, x^{i^*}$ for all $i \in S_k$. By the definition of $\underline{a}_{N\setminus S_k}(R^{i^*}_N, x^{i^*})$ and by the assumption $R^{i^*}_N = R_N$ of the lemma, we must have the fact that $x' = x^{i^*}$ as desired, because it is not the case that $x^{i^*} \, P_i \, x'$ for some $i \in S_k$. □

We next focus on the game (Γ^*, R_N) as a whole and its subgame-perfect equilibria. Three cases of phase 1 are considered in Lemmas 2 through 4.

Lemma 2. *Consider any subgame-perfect equilibrium $m^*_N \in M^*_N$ of (Γ^*, R_N) and its outcome allocation $x \in SPE(\Gamma^*, R_N)$. If case 3 of phase 1 applies on the equilibrium path, then $x \in \varphi(R_N)$.*

Proof. When case 3 of phase 1 applies, at least two agents have chosen positive integers in phase 1. In this case, every agent i has an opportunity to change his report of z^i into an integer that is larger than any other agent's, that is, z^i such that $z^i > z^j$ for all $j \in N \setminus \{i\}$. By reporting such z^i, agent i can reach a subgame that starts at the beginning of round 1 in phase 2–2 as agent i^*. Furthermore, by reporting such z^i together with $R^i_N = R_N$ and $x^i \in \varphi(R_N)$, agent i can realize x^i, which is, by Lemma 1, the subgame-perfect equilibrium allocation of the subgame in phase 2–2. Therefore, every agent i has an opportunity to realize any allocation in $\varphi(R_N)$.

Nevertheless, the equilibrium allocation is x. So, it must be the case that every agent i weakly prefers x to any allocation in $\varphi(R_N)$, that is, for all $i \in N$, we have $x \, R_i \, x'$ for all $x' \in \varphi(R_N)$. Since φ is a sub-correspondence of the β^*-core and any allocation in $\varphi(R_N)$ is a β^*-core allocation, we must have the fact that $x = x'$ for all $x' \in \varphi(R_N)$, that is, $\varphi(R_N) = \{x\}$. □

Lemma 3. *Consider any subgame-perfect equilibrium $m^*_N \in M^*_N$ of (Γ^*, R_N) and its outcome allocation $x \in SPE(\Gamma^*, R_N)$. If case 1 of phase 1 applies on the equilibrium path, then $x \in \varphi(R_N)$.*

Proof. When case 1 of phase 1 applies on the equilibrium path, the play proceeds to phase 2–1 where the β^*-core mechanism Γ is played by all agents. So, the outcome allocation x is realized at some terminal node in Γ. Since the strategy profile $m_N \in M_N$ induced by m^*_N is a subgame-perfect equilibrium for the subgame (Γ, R_N), it must be the case that $g(m_N) = x \in SPE(\Gamma, R_N)$. Since Γ implements φ in subgame-perfect equilibrium, we have the fact that $x \in \varphi(R_N)$. □

Lemma 4. *Consider any subgame-perfect equilibrium $m^*_N \in M^*_N$ of (Γ^*, R_N) and its outcome allocation $x \in SPE(\Gamma^*, R_N)$. If case 2 of phase 1 applies on the equilibrium path, then $x \in \varphi(R_N)$.*

Proof. When case 2 of phase 1 applies, there is only one agent j who has chosen a positive integer in phase 1. In this case, agent j has an opportunity to change his report of $z^j > 0$ into $z^j = 0$ and reach a subgame (Γ, R_N) that is played in phase 2–1. Let $m_N \in M_N$ be the strategy profile induced by m^*_N for the subgame (Γ, R_N), and let x' be the allocation such that $x' = g(m_N)$. Since m_N is a subgame-perfect equilibrium for the subgame (Γ, R_N), it must be the case that $x' \in SPE(\Gamma, R_N)$. Since Γ implements φ in subgame-perfect equilibrium, we have the fact that $x' \in \varphi(R_N)$. Therefore, agent j has an opportunity to realize the allocation $x' \in \varphi(R_N)$. Furthermore, every agent $i \in N \setminus \{j\}$ has an opportunity to realize any allocation in $\varphi(R_N)$ by a similar argument as in the proof of Lemma 2.

Nevertheless, the equilibrium allocation is x. So, it must be the case that every agent i weakly prefers x to $x' \in \varphi(R_N)$, that is, for all $i \in N$, we have $x \, R_i \, x'$. Since x' is a β^*-core allocation, we must have the fact that $x = x'$. □

Lemmas 2 through 4 prove the following proposition.

Proposition 2. *We have the fact that* $SPE(\Gamma^*, R_N) \subset \varphi(R_N)$ *for all* $R_N \in \mathcal{R}_N$.

3–3. Proof: $\varphi(R_N) \subset SPE(\Gamma^*, R_N)$ for all $R_N \in \mathcal{R}_N$.

Choose any arbitrary $R_N \in \mathcal{R}_N$ and fix it throughout this section. Take any $x \in \varphi(R_N)$. We now consider the game (Γ^*, R_N) and its subgames.

We first focus on subgames[17] that start at the beginning of phase 2–1, where the β^*-core mechanism Γ is played by all agents. Recall that at the beginning of Section 3–1, we have been given the mechanism Γ that consists of the set of strategy profiles of all agents M_N, the associated outcome allocation function $g\colon M_N \to X$, and the set of participants at the initial node $T = N$. Since we have the fact that $SPE(\Gamma, R_N) = \varphi(R_N)$ and hence $x \in SPE(\Gamma, R_N)$, we can choose a strategy profile $m_N \in M_N$ for (Γ, R_N) such that $g(m_N) = x$ and m_N is a subgame-perfect equilibrium for (Γ, R_N). We later use this strategy profile m_N to define a strategy profile for (Γ^*, R_N).

We next focus on subgames[18] that start at the beginning of round 1 in phase 2–2. We call each of such subgames *subgame* t where t is the initial node of this subgame, that is, t is the very first node that is reached in phase 2–2. Let S_1^t be the set of non-participants at the node t. Note that each subgame t is a finite extensive game with perfect information, which is played by agents in S_1^t. So, by the theorem due to Kuhn [1953], we can choose a strategy profile $m^t_{S_1^t}$ for subgame t such that $m^t_{S_1^t}$ is a subgame-perfect equilibrium for subgame t. We later use this strategy profile $m^t_{S_1^t}$ to define a strategy profile for (Γ^*, R_N).

We now define a strategy profile $m_N^* \in M_N^*$ for (Γ^*, R_N) as follows.

The strategy profile $m_N^* \in M_N^*$.

Phase 1. For each agent i, his report is such that $(z^i, R_N^i, x^i) = (0, R_N, x)$.

Phase 2–1. For each subgame that starts at the beginning of phase 2–1, the strategy profile induced by m_N^* coincides with m_N.

Phase 2–2. For each subgame t that starts at the beginning of round 1 in phase 2–2, the strategy profile induced by m_N^* coincides with $m^t_{S_1^t}$.

Lemma 5. *The strategy profile* m_N^* *is a subgame-perfect equilibrium for* (Γ^*, R_N) *and its outcome allocation is* x.

[17] We note that there are many such subgames depending on reports $(R_N^i, x^i)_{i \in N}$ in phase 1.
[18] We note that there are many such subgames depending on reports $(z^i, R_N^i, x^i)_{i \in N}$ in phase 1.

Proof. It is easy to see that the outcome allocation for the strategy profile m^*_N is x because for this strategy profile, case 1 of phase 1 applies and the play proceeds to phase 2–1, where the strategy profile m_N is played in the β^*-core mechanism Γ.

We shall show that m^*_N is a subgame-perfect equilibrium for (Γ^*, R_N).

First, for each subgame that starts at the beginning of phase 2–1, the strategy profile induced by m^*_N coincides with m_N, which is a subgame-perfect equilibrium for this subgame.

Next, for each subgame t that starts at the beginning of round 1 in phase 2–2, the strategy profile induced by m^*_N coincides with $m^t_{S^t_1}$, which is a subgame-perfect equilibrium for this subgame.

So, we are left to show that each agent i's report $(z^i, R^i_N, x^i) = (0, R_N, x)$ is his optimal choice in phase 1 given the strategies $m^*_{N\setminus\{i\}}$ of the other agents. Suppose that only agent j deviates unilaterally and chooses $(z^j, R^j_N, x^j) \neq (0, R_N, x)$. If $z^j = 0$, then case 1 of phase 1 applies and the play proceeds to phase 2–1, where the outcome allocation cannot be preferred to $x = g(m_N)$ for agent j. If $z^j \neq 0$, then case 2 of phase 1 applies and the outcome allocation is $x' \equiv h(a^t_j, \underline{a}_{N\setminus\{j\}}(R_N, \underline{x}_j(R_N)))$ where a^t_j is chosen by agent j for his own benefit at the terminal node t. Since $x \in \varphi(R_N)$, we have the fact that $x\, R_j\, \underline{x}_j(R_N)$ by the definition of $\underline{x}_j(R_N)$. Furthermore, we have the fact that $\underline{x}_j(R_N)\, R_j\, x'$ by the definition of $\underline{a}_{N\setminus\{j\}}(R_N, \underline{x}_j(R_N))$. So, we obtain $x\, R_j\, x'$, which means that agent j cannot gain by the deviation such that $z^j \neq 0$. □

Discussions of this section prove the following proposition.

Proposition 3. *We have the fact that* $\varphi(R_N) \subset SPE(\Gamma^*, R_N)$ *for all* $R_N \in \mathcal{R}_N$.

Propositions 2 and 3 complete the proof of the theorem.

4. Conclusion

This paper has pursued a solution to problems of non-participation in a general allocation model. We have proposed how to redesign β^*-core mechanisms to overcome non-participation problems.

Our redesigned mechanisms have the following properties that have been missed in mechanisms in the standard literature. First, agents are given opportunities to show willingness not to participate in β^*-core mechanisms, possibly by passing over such willingness in silence. Second, 'non-participation decisions' are used as messages for the redesigned mechanisms. Third, the redesigned mechanisms respect the rights of non-participants in the sense that the mechanisms never interfere with actions of non-participants. In spite of these properties, the redesigned mechanisms can implement a sub-correspondence of the β^*-core in subgame-perfect equilibrium.

In our redesigned mechanisms, the prefix component consisting of phases 1 and 2–2 plays an important role for inducing participation of agents. The component works as a device for prompting cooperation among agents.

However, the component works properly only for β^*-core mechanisms. It is still unknown whether there is a way to induce participation of all agents in mechanisms that aim to attain allocations not in the β^*-core.

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