Persistence and Volatility of Hedge Fund Returns: ARMA-GARCH Modeling

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1. Introduction

For over two decades hedge funds have been focused the world's attention on their tremendous growth. At the same time, the international financial community has expressed serious concern about whether they have played a crucial role in triggering financial crises. They have also been attracting the attention of institutional investors such as pension funds since the IT bubble burst in 2003. One of the main reasons for such interest stems from the peculiar performance characteristics of the hedge fund sector. Hedge fund managers employ frequently dynamic trading strategies involving short sales, leverage and derivatives, and thus, they tend to generate returns less uncorrelated to those of market benchmark returns.

Hedge funds are now major market participants and they are no longer preceived as mavericks in global financial markets. Their dynamic, multi-faceted investment strategies have now penetrated publically traded ETFs. Investable hedge fund indices are really regarded as the disguise of funds of hedge funds (Jaeger [2008]). For example, investable hedge fund indices tracking the performances of their strategies are used as "index" funds, whose purpose is "hedge fund replication" for institutional investors. Replicating hedge fund returns means replicating their return sources and corresponding risk exposures based on their strategies. The 2008 financial crisis has significantly decreased the returns of most hedge fund strategies. Many market participants in the hedge fund industry realized there is no safe place for investors to avoid systematic risk, and questioned whether diversification across hedge funds as an alternative investment is really as beneficial as they intended. Therefore, investors who aim to put money into investable hedge fund indices must understand their return sources to achieve replication.

Univariate time-series data of hedge fund returns themselves exhibit peculiar characteristics of non-normal distribution such as heavy-tailed and skewed distribution, and volatility clustering. Volatility is one of the most important concepts of finance. It is often regarded as a measure of financial risk, calculated by the variance or standard deviation of an asset's return. It is well known that there are some periods of high volatility and other periods of low volatility of asset returns in financial markets. Volatility clustering implies that volatility shocks today will influence the expectation of volatility many periods in the future. This phenomenon requires analysts to describe returns and volatility that are nonlinear.

Volatility is not directly observable in the financial market, such as in stock prices. It is described as a parameter of the stochastic processes that is applied to model variations in financial asset prices. It is only quantifiable in the context of a model, and thus, the results of the estimates can be quite different depending on the model and on the market conditions. Many studies have argued that nonlinear processes model the volatility behavior of hedge fund strategies better (Füss, R., D. G. Kaiser and Z. Adams [2007], Blazsek, S. and A. Downarowicz [2011], Del Brio, E. B., A. Mora-Valencia and J. Perote [2014], Teulon, F., K. Guesmi and S. Jebri [2014]). In the context of portfolio diversification, including hedge funds, precise volatility modeling of hedge fund returns may help institutional investors to evaluate the future risk of hedge fund portfolio and are useful to determine market timing and control the risk limit.

The purpose of this paper is to examine the conditional volatility characteristics of daily management hedge fund index returns and construct an ARMA-GARCH type modeling. This paper will limit itself to the univariate time-series analysis of hedge fund returns although the issues studied here will be similar in multivariate analysis. I focus on the construction of nonlinear time-series models that can be useful for describing persistence and volatility of hedge fund index returns. This paper is organized as follows. Section 2 describes four main hedge fund strategies and summarizes the empirical properties of their return series used in this study. Section 3 reviews ARMA modeling and presents the estimation results and diagnostic checking. In Section 4, GARCH modeling is introduced and discusses the results. Some concluding remarks are offered in the final section.

2. Hedge Fund Strategies and Data Description

In this paper, four principal hedge fund strategies indices (Equity Hedge, Event Driven, Macro/CTA, and Relative Value Arbitrage in the HFRX Global Hedge Fund Index) are investigated. Data are daily and span the period March 31, 2003 to August 11, 2014. The data of hedge fund indices is obtained from the Hedge Fund Research Inc. (hereafter HFR). The HFRX Global Hedge Fund Index is designed to be representative of the overall composition of the hedge fund universe and to be investable.[1] It is comprised of all eligible

1) HFRX Hedge Fund Indices are the global industry standard for performance measurement across all aspects of the hedge fund industry. Constituents of all indices are selected from an eligible pool of the more than 6,800 funds that report of the HFR Database. More detailed strategy descriptions can be seen in Hedge Fund Research [2014], *HFRX Hedge Fund Indices: Defined Formulaic Methodology*<www.hedgefundresearch.com>.

hedge fund strategies falling within these four principal strategies. First, Equity Hedge is the strategy maintaining long and short in primarily equity and equity derivative securities. Its investment decision includes both quantitative and fundamental techniques; broadly diversified strategies or narrowly focused on specific sectors, and frequently employed leverage. Equity Hedge is the directional strategy. Second, Event Driven is the strategy that focuses specifically on corporations involved in special situations or significant restructuring events such as mergers, liquidations and insolvencies. The goal of this strategy is to take advantage of price anomalies triggered by special events. Securities include a variety of types from most senior in the capital structure to most junior or subordinated, and frequently involve additional derivative securities. Event Driven is categorized as the non-directional and mispricing strategy. Third, Macro is the directional strategy based on the prediction to future macroeconomic movements, whose managers employ a variety of techniques. Fourth, Relative Value Arbitrage is the arbitrage strategy that tries to take advantage of temporarily mispricing valuations in the relationship between multiple securities. The security type involves the broad range across equity, fixed income, derivative or other security types. Relative Value Arbitrage is the non-directional strategy.

Figure 1 plots daily index values (upper panel) and returns (lower panel) of (a) Equity Hedge, (b) Event Driven, (c) Macro/CTA and (d) Relative Value Arbitrage. It offers a first look at the data by showing a selection of the index values and the corresponding logarithmic returns measured in percentage terms. Index returns are calculated as continuous compounded returns, defined as $r_t = \log(p_t/p_{t-1})*100$ where p_t denotes the corresponding index value over the sample period. It is easy to see the steady growth of all index values before the subprime crisis and the subsequent sharp decline after the Lehman shock in 2008. The return series (i.e. daily price changes) are centered around zero throughout the sample period. One of the most important features of these return series is that the amplitude of the returns is changing. The magnitude of the changes is sometimes large and sometimes small, that is, it displays time-varying volatility, which is known as volatility clustering. Volatility clustering stems from positive autocorrelation coefficients of squared returns. The technical term applied to this phenomenon is autoregressive conditional heteroscedasticity. From a different viewpoint, volatility measured by squared returns is persistent, hence to some extent predictable.

Table 1 reports summary statistics for the daily returns of four hedge fund indices. The performance statistics suggest the following points. First, all hedge fund indices indicate that the unconditional probability distributions of their returns are leptokurtic. Leptokurtosis implies more weight in both tails of the distribution than in the normal distribution, which indicates a 'fat tailed' distribution. This outcome means that large negative and positive returns are much more likely than would be the case under a normal distribution for these indices. Those return distributions show evidences of fat tails and a higher peak.

Figure 1: Four Hedge Fund Index Returns from April 1, 2003 to August 11, 2014

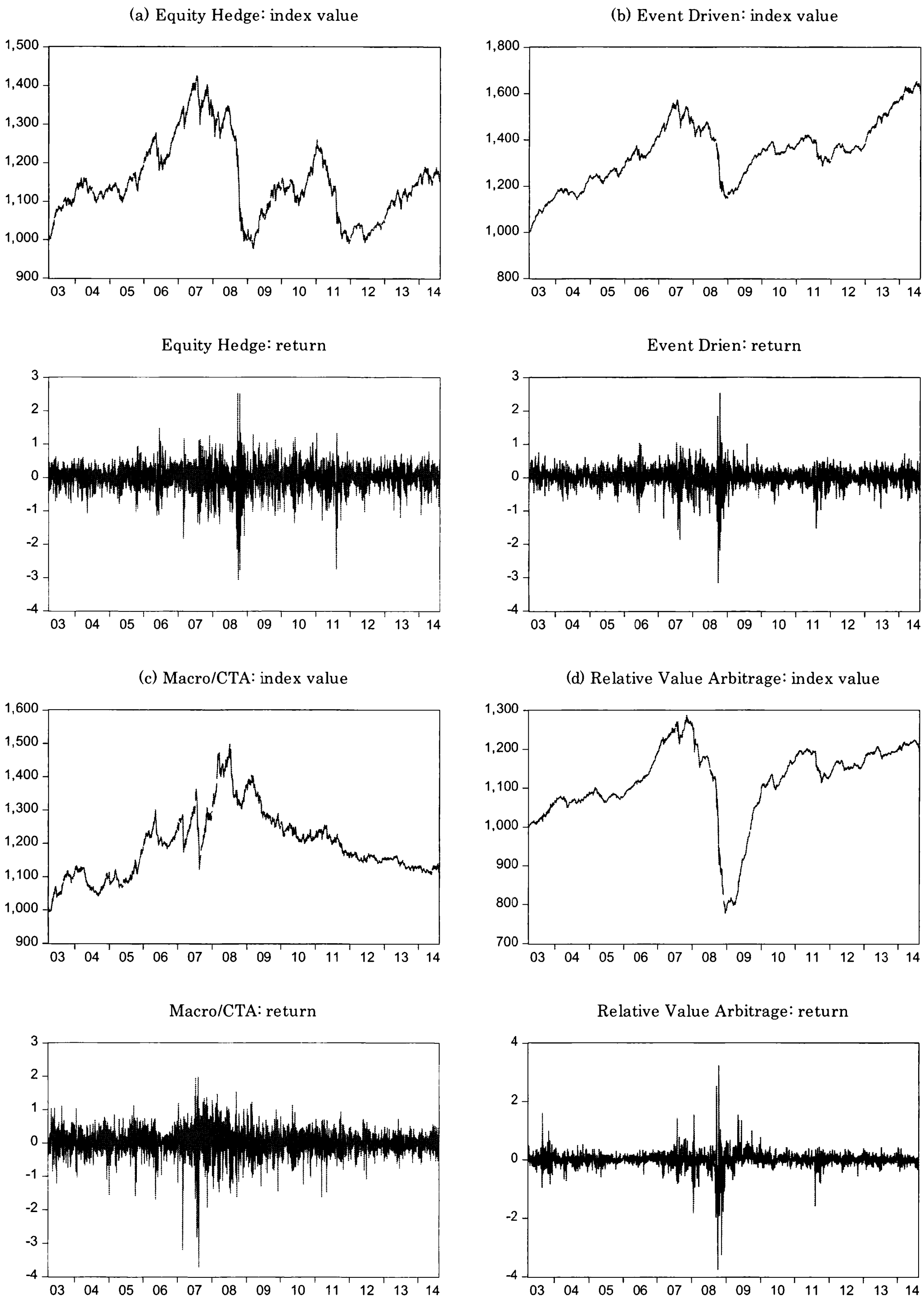


Table 1: Summary Statistics of Hedge Fund Index Returns

April 1, 2003 to August 11, 2014

Daily Return	Mean	STD	Skewness	Kurtosis	Jarque -Bera	No.Obs.
HFRX Global Hedge Fund Index						
Equity Hedge	0.0052	0.4066	-0.8442	8.6599	4162.95***	2864
Event Driven	0.0171	0.2959	-1.1558	15.0343	17919.96***	2864
Macro/CTA	0.0039	0.4081	-1.0193	10.5510	7300.02***	2864
Relative Value Arbitrage	0.0065	0.2712	-1.7268	41.7891	180971.40***	2864

Source: Author's calculations, based on data from Hedge Fund Research.
Notes: The Jarque-Bera normality test is asymptotically distributed as a central χ^2 with 2 degrees of freedom under the null hypothesis, with 10%, 5% and 1% critical values. *, **, *** denote significance at the 10%, 5%, and 1% levels, respectively.

Second, all hedge fund return distributions are negatively skewed. Negative skewness means that the left tail is particularly extreme. It indicates that large negative returns are more probable than large positive ones. Negative skewness and leptokurtosis are unattractive features for risk-averse investors.

The statistical properties of non-normally distributed hedge fund index returns pose difficult problems for measuring risk. The standard deviations imply average daily volatilities, often used as a risk measurement. However, it can only be appropriate for a risk if the observed returns are normally distributed. Traditional risk management based on the mean-variance approach only takes two parameters-mean return and return variance (and/or standard deviation)-into account to specify the risk-return profile of the investor's portfolio. If the returns are normally distributed, the first two moments of the distributions are enough to characterize their risk-return profile. However, in the case of non-normally distributed returns, skewness and kurtosis might play a significant role on risk perception for investors. As is evidenced by their significant JB-test statistics, it seems appropriate to conclude that all hedge fund index returns are not normally distributed.

3. ARMA Modeling: Linear Structure in Univariate Time Series

The univariate time-series of our interest is the hedge fund index value p_t at time t. Any time-series data, p_t such as financial asset prices can be thought of as random variables having been generated by a stochastic process. A concrete set of data, p_t, p_{t+1}, p_{t+2}, $\cdots$ can be regarded as a particular realization of the underlying stochastic process (i.e. the values of a random variables).

In time series regression, the idea that historical relationships (i.e. the future is like the past) can be generalized to the future is formalized by the concept of stationarity. The perception that the future will be like the past is an important assumption in time series regression, so much so that it is given its own name, "stationarity". It is well known that, in most financial time series, prices are non-stationary while the returns are stationary. To confirm this for four hedge fund index returns, the unit root tests are used in detecting whether the returns series are stationary or nonstationary. According to the unit root tests (the augumented Dicky-Fuller test and the Phillip-Perron test) for the null hypothesis that the series has a unit root (i.e. it is nonstationary), all index returns can reject the null hypothesis for significance at 99% confidence levels, which means stationarity for those series.[2)]

With time-series data, it is likely that the observations will be correlated over time because the observation at time t is the consequences of economic actions or decisions taken at time t, but also at time t+1, t+2, and so on. As shown in Figure 2, these effects do not occur instantaneously but are spread over future time periods.

A popular method of modeling stationary time series is the autoregressive moving average (ARMA) method which assembles two separate tools (AR terms and MA terms) for modeling the serial correlation in the lagged dependent variable and in the disturbance. It can be saying that the dependent variable r_t in one period will depend on what it was in the past periods, r_{t-1}, r_{t-2}, $\cdots$, which is the persistence of hedge fund performance over various time intervals. Another way of modeling the continuing impact of change over several periods is via the error term, which represents the composition of all factors (apart from the independent variables) that influence the behavior of the dependent variable. The behavior of these factors in the current time period might be quite similar to their behavior in the previous time period and suggests the possibility of some correlation between errors close together in time.

In this section, these two ways in which dynamics can enter regression relationship-lagged values of the dependent variable (AR terms), and lagged values of the error term (MA terms) are considered.

First, consider the unconditional moments of the return process. The mean μ is defined as

$$\mu = \mathrm{E}[r_t] \tag{1}$$

where E[•] denotes the expectation operator and the expected value of the return (i.e. the expected return) $\mathrm{E}[r_t]$. The variance of r_t is a measure of dispersion in the possible values for r_t, denoted as $\mathrm{var}(r_t)$, is defined as

$$var[r_t] = E[r_t - \mu]^2 = \sigma^2 \tag{2}$$

where its square root σ is the standard deviation of r_t, which is called volatility and a measure of risk.

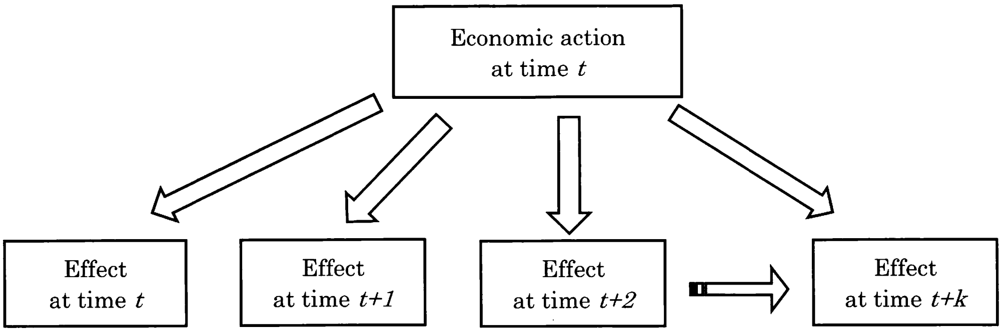
In general, the return on any asset r_t can be divided into two parts: the expected parts of the return $\mathrm{E}[r_t]$ and the unexpected part of the return ε_t.

$$r_t = \mathrm{E}[r_t] + \varepsilon_t \tag{3}$$

$$r_t = \mu + \varepsilon_t \tag{4}$$

2) The distribution theory supporting the Dickey-Fuller test assumes that the disturbance terms are uncorrelated and homogeneous. The augumented Dickey-Fuller test allows the disturbance terms are correlated but still assume to be homogeneous. Moreover, the Phillip-Perron test allows the disturbance terms to be correlated and heterogeneously distributed. See Enders [1995], p.239.

Figure 2: The Distributed Lag Effect



Source: Author's compilation based on Griffiths, Hill and Lim [2008], p.227.

where ε_t, is known as the disturbance, or error term.

The error term is a random variable that has the probabilistic properties with zero mean, constant variance and serially uncorrelated. Such error term is called a white noise error term, which is defined by

$$E[\varepsilon_t] = 0 \tag{5}$$

$$E[\varepsilon_t^2] = \sigma^2 \tag{6}$$

$$E[\varepsilon_t \varepsilon_s] = 0 \qquad for\, s \neq t. \tag{7}$$

In the context of financial analysis, the errors ε_t are often considered as "shocks" or "news". They represent unexpected factors. Then, equation (3) implies that an observed time series r_t is related to an underlying sequence of shocks ε_t.

The predictable component of r_t is often formulated as an autoregressive process since a time series variable often relates to its past values in many cases.

$$r_t = \emptyset_1 r_{t-1} + \emptyset_2 r_{t-2} + \cdots + \emptyset_p r_{t-p} + \varepsilon_t, \qquad t = 1, \ldots, n \tag{8}$$

$$r_t = \sum_{i=1}^{p} \emptyset_i \, r_{t-p} + \varepsilon_t \tag{9}$$

where $\emptyset_1, \ldots, \emptyset_p$ are the values of the parameters which measure the impact of the previous return, lies between -1 to 1. This simple model is called an autoregressive model of order *p*.

The MA part of the model refers to the structure of the error term. The first-order moving average model, MA(1), is

$$r_t = \mu + \varepsilon_t + \theta_1 \varepsilon_{t-1} \tag{10}$$

where $\emptyset_1$ scales the influence of the white noise process.

The MA(q) process can be written as

$$r_t = \mu + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \cdots + \theta_q \varepsilon_{t-q} \tag{11}$$

Equation (11) states that a moving average model is simply a linear condition of white noise process. In other words, r_t depends on the current and previous values of a white noise error term. More concisely,

$$r_t = \mu + \sum_{j=1}^{q} \theta_j \, \varepsilon_{t-j} \tag{12}$$

By combining the AR(p) and MA(q) models, an ARMA(p,q) model is obtained as follows.

$$r_t = \mu + \emptyset_1 r_{t-1} + \emptyset_2 r_{t-2} + \cdots + \emptyset_p r_{t-p} + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \cdots + \theta_q \varepsilon_{t-q} \tag{13}$$

Equation (13) states that the current value of returns series r_t depends linearly on its own pervious values plus a combination of current and previous values of a white noise error terms. Namely, the autoregressive and moving average specifications can be combined to form an ARMA(p,q) model.

Model identification

The strategy of an appropriate ARMA model selection is systematic, i.e. the so-called Box-Jenkins approach. This approach takes three steps: identification, estimation and diagnostic checking.

The first step of building an ARMA model is to identify the order of the model required to capture the features of data generating process. It is to determine the appropriate AR and MA orders p and q. A central concern of this approach is to specify for the predictable part as a constant μ and measure the error term ε_t, which is the difference of the series from its mean $r_t - \mu$ as shown in equations (4).

Identification of the structure in the data is carried out by looking at the autocorrelation and partial autocorrelation coefficients after plotting the data over time. Autocorrelation is the correlation of a series with its own lagged values. When the observations in different time periods are correlated, it is said that autocorrelation exists. The coefficient of correlation between the observations at two adjacent periods is called the autocorrelation coefficient. Table 2 displays the autocorrelation function (ACF) of the hedge fund index returns. The estimated autocorrelation coefficients for lag 1 to 20 together with the Ljung-Box (LB) statistics with five, ten and twenty autocorrelations are reported. At first glance, the ACF of the return series show that there is a slightly autoregressive structure in the data. In particular, Relative Value Arbitrage shows highly significant autocorrelations over all lags. Thus, it seems that either an AR or a mixed ARMA process might be appropriate for modeling these data. In fact, it is not easy to precisely determine the appropriate lag order given these estimates at this stage.

It is possible to test the joint hypothesis that all of the first m (= maximum lag length) autocorrelation coefficients are simultaneously (jointly) equal to zero (H_0: $\rho_1 = \rho_1 = \cdots = \rho_m = 0$). Q-statistics is the Ljung and Box statistic of ACF (LB-Q), represented in the bottom part of Table 2. The returns of four indices excepting for Relative Value Arbitrage do not show high autocorrelation coefficients, but some of them are still highly significant at 95% confidence level. Since the first ACF coefficients of all returns series are highly significant, the Ljung-Box joint test statistic rejects the null hypothesis of no autocorrelation at the 1% level.

Table2: Autocorrelations

ACF	Equity Hedge	Event Driven	Macro/CTA	Relative Value Arbitrage
Lag(1)	0.154***	0.108***	0.104***	0.195***
Lag(2)	0.027	0.060***	0.031*	0.107***
Lag(3)	0.031*	0.078***	0.034*	0.124***
Lag(4)	0.013	0.016	0.040**	0.125***
Lag(5)	-0.014	0.066***	-0.004	0.094***
Lag(6)	-0.017	0.006	-0.017	0.071***
Lag(7)	0.017	0.022	0.009	0.100***
Lag(8)	0.021	0.039**	0.028	0.092***
Lag(9)	0.028	0.049***	0.013	0.105***
Lag(10)	0.050***	0.032	0.031*	0.094***
Lag(11)	0.001	0.018	0.001	0.041**
Lag(12)	0.021	0.058***	0.028	0.172***
Lag(13)	0.021	0.024	0.016	0.151***
Lag(14)	0.005	0.038**	0.024	0.069***
Lag(15)	0.006	0.024	-0.002	0.140***
Lag(16)	0.067***	0.084***	0.000	0.175***
Lag(17)	0.027	0.035*	-0.021	0.117***
Lag(18)	-0.047**	-0.024	-0.009	0.054***
Lag(19)	0.039**	0.015	0.052***	0.070***
Lag(20)	0.018	0.024	-0.013	0.074***
LB-Q(5)	73.658***	74.326***	41.358***	255.97***
LB-Q(10)	85.902***	89.994***	47.886***	380.05***
LB-Q(20)	115.3***	135.61***	62.409***	771.68***

Source:Author's calculations, based on data from Hedge Fund Research

Note:The significance tests for the autocorrelation coefficients can be constructed by a non-rejection region for an estimated autocorrelation coefficient to determine whether it is significantly different from zero. Under the assumption that returns are normally distributed, confidence intervals for the correlations can be constructed.

For a sample size of T, a correlation coefficient is defined as statistically significant at the 10%, 5% and 1% levels would be given by $\pm 1.65/\sqrt{T}$, $\pm 1.96/\sqrt{T}$ and $\pm 2.58/\sqrt{T}$, respectively. *, ** and *** denote significance at the 10%, 5%, and 1% levels, respectively.

A way of deciding on the appropriate model orders is to use an information criterion. There are two popular information criteria: Akaike information criterion (AIC) and Schwarz's (Bayesian) information criterion (SIC).[3] The approach to choosing numbers of lags, p, q, in ARMA model is to estimate them by

3) These criteria compare the in-sample fit, which is measured by the residual variance $\hat{\varepsilon}^2$, against the number of estimated parameters k. In more detailed explanation about these information criteria, see Franses and van Dijik [2000], p.38 and Brooks [2008], p.233.

minimizing information criteria.

Let T and k denote the sample size and the total number of parameters in the ARMA(p,q) model, that is, $k = p + q + 1$. First, AIC is computed as

$$\begin{aligned} \text{AIC}(k) &= T\ln\hat{\sigma}^2 + 2k \\ &= \ln\hat{\sigma}^2 + \frac{2k}{T} \end{aligned} \qquad (14)$$

where $\hat{\sigma}^2 = 1/T\sum_{t=1}^{T}\hat{\varepsilon}_t^2$ with $\hat{\varepsilon}_t$ the residuals from the ARMA model.

Second, SIC is computed as

$$\begin{aligned} \text{SIC}(k) &= T\ln\hat{\sigma}^2 + k\,ln\,T \\ &= \ln\hat{\sigma}^2 + \frac{k}{T}\,ln\,T \end{aligned} \qquad (15)$$

The difference between the AIC and the SIC is the second term. " $\ln T$ " of the second term in the SIC is replaced by "2" in the AIC. Because $\ln T > 2$ for $T > 8$, the SIC penalizes additional parameters more heavily than the AIC. For example, for the 2864 observations of the returns series under investigation used to estimate the ARMA modeling, $\ln(T) = \ln(2864) \approx 7.960$, so the second term for the SIC is almost four times as large the term in AIC. Therefore, the model order selected by the SIC is likely to be smaller than that selected by the AIC.

A natural question to ask of any estimated model is: Which criterion should be preferred if AIC and SIC suggest different model orders? The principle of parsimony is based on the Box-Jenkins approach. A parsimonious model is the model that describes all of the features of data of interest using as few parameters (i.e. as simple a model) as possible.[4] It fits the data well without incorporating any needless coefficients. In large samples, the AIC will overestimate the number of lags with nonzero probability since the second term is not large enough to ensure that the correct lag length is chosen, so the AIC estimator is not consistent.[5] On the other hand, the improvement in fit caused by increasing the AR and /or MA orders needs to be quite substantial for the SIC to favor a more elaborate model. Franses and van Dijk [2000] point out that in practice, the SIC prefers very parsimonious models, containing only few parameters.

Parameter estimation and residual diagnostics

ARMA modeling of univariate time-series data is not based on any economic or financial theory. The purpose of constructing these models is to capture relevant features of the observed data under consideration.

4) A parsimonious model is desirable since the residual sum of squares is inversely proportional to the number of degree of freedom. A model that contains irrelevant lags of the variable or of the error term (and therefore unnecessary parameters) will usually lead to increased coefficient standard errors, implying that it will be more difficult to find significant relationships in the data. See Enders [1995] p.95. and Brooks, [2008], pp. 231-232.

5) Stock and Watson [2012] explain the details of this point, in Appendix 14.5. " Consistency of the BIC Lag Length Estimator" (pp.623-624).

Thus, the estimated output of an ARMA model may be better to understand the plausibility of the model as a whole and to determine whether it exhibits the properties of the data well, and consequently provides accurate forecasts.

Table 3 shows the estimated ARMA process for four index returns selected by the SIC criterion. For an ARMA model, a set of statistics of the estimated AR and MA parameters are the serial correlation coefficients of the lagged dependent and disturbance variables, in which the values lies between -1 (extreme negative serial correlation) and +1 (extreme positive serial correlation).

Before applying the selected ARMA models for index returns series, it is necessary to look for signs of model misspecification. Here the procedure for testing the adequacy of an estimated ARMA model is to investigate whether the estimated residual series $\hat{\varepsilon}_t$ is approximately white noise. First of all, it is particularly important that the residuals from an estimated model be serially uncorrelated. Any evidence of autocorrelation implies a systematic movement in the sequence of r_t that is not accounted for by the ARMA coefficients included in the model. In the case of given model adequacy, the error term would be a white noise process with no autocorrelation as shown in equation (7). Therefore, after fitting candidate ARMA specifications, we should verify that there is no autocorrelation in the residuals of the models. So, I begin by examining whether or not there are autocorrelation in the error term of an estimated ARMA model. Autocorrelation diagnostic tests for the residuals were computed to check the adequacy of the estimated ARMA models by using the Breusch-Godfrey serial correlation LM test for higher order ARMA errors. The null hypothesis is that there is no serial correlation up to the rth order.[6] In other words, the errors are uncorrelated with one another. The observed R-squared statistics is the Breusch-Godfrey LM test statistic and its estimated value (see Serial Cor. LM test) is shown in Table 3. If the test statistic exceeds the critical value from the Chi-squared statistical tables, the null hypothesis of no autocorrelation can be rejected. The null hypothesis of no autocorrelation cannot be rejecled by any of the estimated ARMA models,which means that the models satisfy the assumption that the covariance between the error terms over time is zero, $\mathrm{cov}(\varepsilon_i, \varepsilon_j) = 0\ for\ i \neq j$.

The estimated ARMA processes for Event Driven and Relative Value Arbitrage exhibit statistically significant large positive values of coefficients of the AR(1) terms and large negative values of coefficients of the MA(1) terms. These estimates indicate that the returns to the nondirectional strategies of Event Driven

6) The simplest test of detecting autocorrelation is the Durbin-Watson test, which is a test for first-order autocorrelation. Therefore, Durbin-Watson statistics can verify the null hypothesis of no autocorrelation against the alternative hypothesis of first-order autocorrelation. In addition, the DW test is no longer valid if there are lagged dependent variables on the right-hand side of regression such as AR models. The Breusch-Godfrey serial correlation LM test is a more general test for autocorrelation. For more detailed technical discussion about detecting autocorrelation, see Brooks [2008], pp.143-150.

Table 3: ARMA Modeling

Indices	Equity Hedge	Event Driven	Macro /CTA	Relative Value Arbitrage
Model	AR(1)	ARMA(1,2)	AR(1)	ARMA(1,2)
Parameter estimation				
$\hat{\mu}$	0.0051	0.0145	0.0039	0.0049
	(0.0090)	(0.0120)	(0.0088)	(0.0193)
$\hat{\Phi}_1$	0.1539***	0.9731***	0.1038***	0.9818***
	(0.0223)	(0.0154)	(0.0319)	(0.0128)
$\hat{\theta}_1$	—	-0.8824***	—	-0.8442***
		(0.0303)		(0.0545)
$\hat{\theta}_2$	—	-0.0612**	—	-0.0846
		(0.0267)		(0.0522)
SIC	1.0194	0.3887	1.0400	0.1534
Diagnostic checking				
Autocorrelation: $\hat{\varepsilon}$ Serial Cor. LM test	0.0736	0.4824	1.1747	0.4735
Normality: $\hat{\varepsilon}$ Jarque-Bera	4536.585***	17401.09***	6330.741***	246589***
ARCH effect: $\hat{\varepsilon}^2$ ARCH LM(1) test	135.081***	83.228***	163.198***	43.391***

Notes: Based on daily continuously compounded returns from 04/01/2003 to 08/11/2014; standard errors are presented in parenthesis; The statistical significance is determined by using HAC autocorrelation-heteroscedasticity -consistent standard errors (Newey-West); ***, **, * denote significance at 99%, 95% and 90% confidence levels, respectively.

and Relative Value Arbitrage have high serial correlation. The most likely explanation is that the indices to these hedge fund strategies involve less liquid assets. On the contrary, the directional strategies such as Equity Hedge and Macro/CTA exhibit relatively low serial correlation.

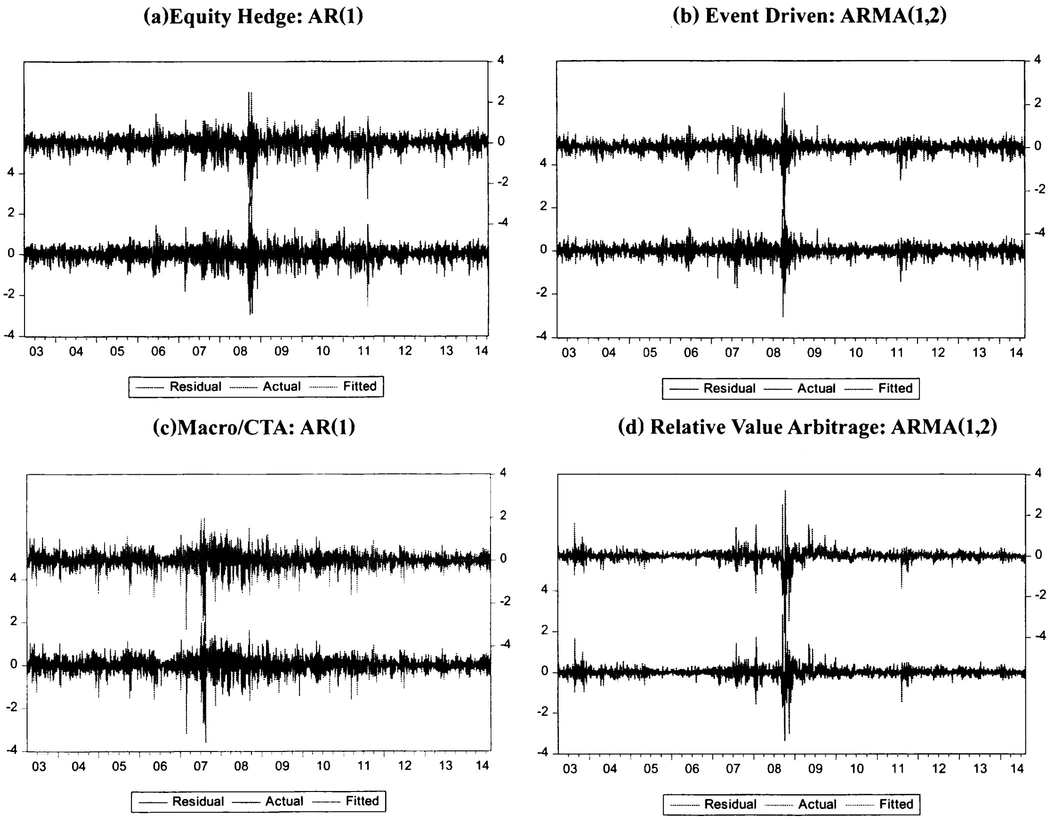
To get a feel for the fit of the residuals in the models, the residual graphs are depicted in Figure 3. The actual and fitted values are depicted on the upper portion of the graph. The lower portion depicts the difference between the actual and fitted values, which provides little control over the process of producing fitted values. It seems obvious that the residuals of the ARMA models have systematically changing over the sample period, that is, a sign of heteroscedasticity.

In linear time series models the errors ε_t, in other words, the underlying shocks are assumed to be uncorrelated but not necessarily to be independently identically distributed (IID).[7)]

$$\varepsilon_t \sim IID\, N(0, \sigma^2) \quad (16)$$

7) See Campbell, Lo and MacKinlay [1997], p.468.

Figure 3: Residual Graphs of Estimated Models



where ε_t is independently, identically and normally distributed with a zero mean and a constant variance σ^2. The white noise time series process exhibits no trends or clusters since the observations are independent each other.

In nonlinear time series models the underlying shocks are typically assumed to be IID. Before proceeding to the next step for nonlinear models that is sufficient to describe these important features of the data, the assumption to be IID normal in the error term should be examined to the residuals. One of the most popular tests for normality is the Jarque-Bera test. According to Jarque-Bera test statistics, any hypothesis that the residuals of all models are normally distributed can be rejected. In financial modeling, one or two outliers cause a rejection of the normality assumption. Outliers also appear in the tails of the distribution, which enters into the value of kurutosis to be large as shown in Table 1.

White noise error term is assumed to be homoscedastic. The expected value of all error terms, when squared, is the same at any given point. This assumption is called homoscedasticity. Time series data in which the variance of the error terms are not equal, in which the error terms may reasonably be expected to be larger or smaller for some points, are said to be heteroscedastic. Figure 4 depicts the squared residuals of the estimated ARMA models, in which the error variances are time varying. In addition, the squared residuals are serially correlated due to their correlograms and autocorrelation coefficients.

The heteroscedasticity specification is examined by the ARCH (autoregressive conditional heteroscedasticity) effects in the residuals. The ARCH LM test investigates whether the magnitude of residuals appeared to be related to the magnitude of recent residuals. To test for first order ARCH, regress the squared regression residuals

$$\hat{\varepsilon}_t^2 = \gamma_0 + \gamma_1 \hat{\varepsilon}_{t-1}^2 + v_t \quad (17)$$

where v_t is a random term. The null and alternative hypotheses are:

$H_0: \gamma_1 = 0$

$H_1: \gamma_1 \neq 0$

If there are ARCH effects, the magnitude of $\hat{\varepsilon}_t^2$ depends on its lagged values. More generally, the null hypothesis is that there is no ARCH up to order q in the residuals $\hat{\varepsilon}_t^2$. The test can be thought of as a test for autocorrelation in the squared residuals. The ARCH LM test is implemented to make sure that this class of models is appropriate for the data under investigation before estimating a GARCH-type model.

The outputs of ARCH LM tests for ARCH (1) effects, in which the number of lags to include is 1 are indicated in Table 3. The null hypothesis that there are no first order ARCH effects can be rejected for all index returns since the LM-statistics are very significant. It suggests that the presence of ARCH effects in the squared residuals of the models.

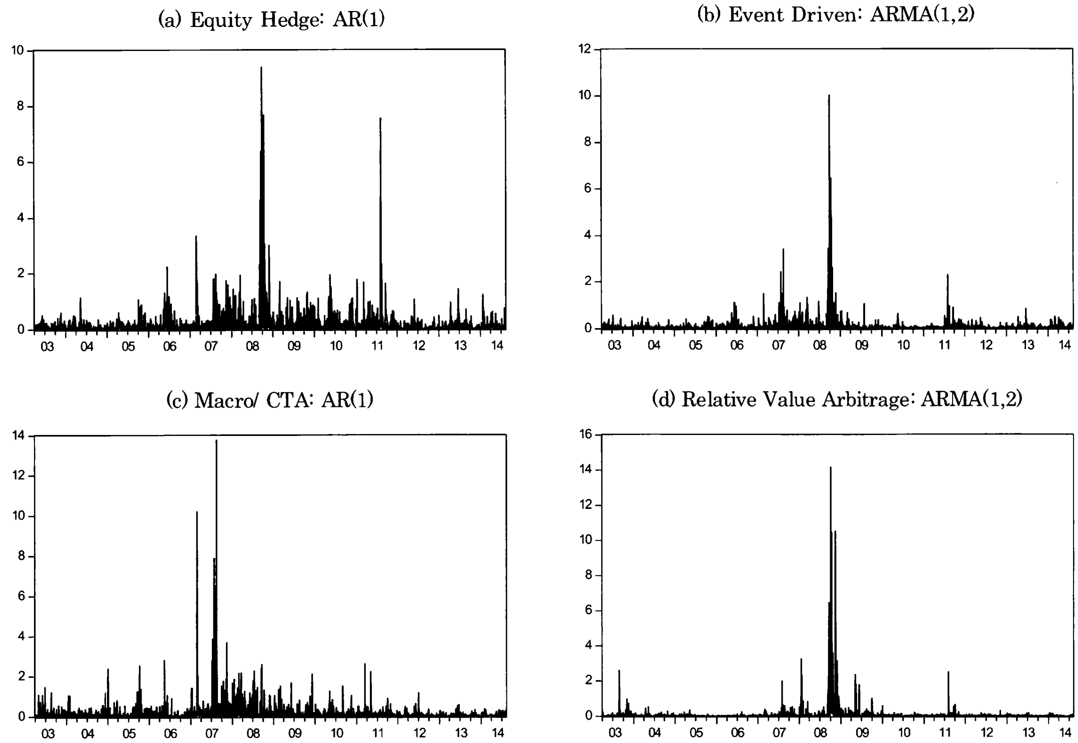
4. GARCH Modeling

So far, I have focused on ARMA modeling for the predictable part of the return $\mathrm{E}[r_t]$ or μ. After diagnostic testing for residuals, it is clear that the residuals ε_t are not autocorrelated, but they are not independently, identically normally distributed or varying over time.

In the context of time-series analysis, the defined unconditional moments in section 3 have referred to the long-run moments of the series, that is, the unconditional mean, variance and covariance at $t \to \infty$. In addition to the long-rum unconditional moments, the conditional moments can be defined.

The expected parts of the return is what can be predicted using the knowledge from the past, which is denoted by Ω_{t-1} the information set of all available information up to and including time $t-1$. This expected part of the return is conditional mean $\mathrm{E}[r_t|\Omega_{t-1}]$, which is the mean at time t conditional on the information set taken by the series in previous periods.

Figure 4: Squared residuals



$$r_t = \mathrm{E}[r_t|\Omega_{t-1}] + \varepsilon_t \tag{18}$$

where $\mathrm{E}[\cdot|\cdot]$ denotes the conditional expectation operator, and thus the conditional mean is $\mathrm{E}[r_t|\Omega_{t-1}] = \mu$.

The white noise error term can also be defined as its conditional mean, variance and covariance.

$$\mathrm{E}[\varepsilon_t] = \mathrm{E}[\varepsilon_t|\Omega_{t-1}] = 0 \tag{19}$$

$$\mathrm{E}[\varepsilon_t^2] = \mathrm{E}[\varepsilon_t^2|\Omega_{t-1}] = var(\varepsilon_t) = \sigma^2 \qquad \text{for all } t \tag{20}$$

$$\mathrm{E}[\varepsilon_t\varepsilon_s] = cov(\varepsilon_t, \varepsilon_s) = 0 \qquad \forall\, s \neq t. \tag{21}$$

The important point to note here is that the variance of ε_t is assumed to be both unconditionally and conditionally homoscedastic – that is, $\mathrm{E}[\varepsilon_t^2] = E[\varepsilon_t^2|\Omega_{t-1}] = \sigma^2$ for all t in equation (20).

Researchers or traders engaged in forecasting asset prices have often experienced that their ability to forecast asset prices varies, consequently their returns vary considerably from time to time. For one time period the forecast errors might be relatively small, while they might be relatively large for another period. This variability could very well depend on volatility in financial markets.

The forecast error is considered as

$$\hat{r}_t - \mathrm{E}[r_t|\Omega_{t-1}] = \hat{\varepsilon}_t \tag{22}$$

From this viewpoint, it seems sensible to explain volatility as a function of the error term ε_t . This would suggest that variance of forecast errors is not constant but varies from time to time.

Generalized autoregressive conditional heteroscedastic (GARCH) Model

Now I move to relax the assumption of homoscedasticity. When the error terms do not all have the same variance, they are said to exhibit heteroscedasticity, which allows the conditional variance of ε_t to vary overtime, that is, time-varying volatility (Figure 5). The estimated error $\hat{\varepsilon}_t$ is the difference between the observed and predicted values, given by $\hat{\varepsilon}_t = r_t - \hat{r}_t$. It can be used to obtain the estimated conditional variance h_t. The conditional variance can estimate the variance of a series at a particular point in time *t*.

$$E[\varepsilon_t^2|\Omega_{t-1}] = \sigma_t^2 = h_t \tag{23}$$

Equation (23) states that the new information set Ω at time t -1 is captured by the most recent squared residual. Such an updating rule is a simple description of adaptive or learning behavior and might be expressed as a kind of autocorrelation in the variance of forecast errors. To capture this serial correlation of volatility, Engle, R. F. [1982] developed the autoregressive conditional heteroscedasticity (ARCH) model.[8)] The key idea of the ARCH model is that the variance of ε at time *t*, that is, $h_t(= \sigma_t^2)$ depends on the size of the squared error term at the previous time $t-1$, that is, on ε_{t-1}^2.

$$h_t = \sigma_t^2 = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 \tag{24}$$

where the conditional variance depends on only one lagged squared error. This is called an ARCH(1) process. In general, an ARCH(q) model that includes lags $\varepsilon_{t-1}^2, \dots, \varepsilon_{t-q}^2$ has a conditional variance function that is given by

$$h_t = \sigma_t^2 = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \alpha_2\varepsilon_{t-2}^2 \cdots + \alpha_q\varepsilon_{t-q}^2 \tag{25}$$

If there is no autocorrelation in the error variance, the null hypothesis, $H_0\colon \alpha_1 = \alpha_2 = \cdots = \alpha_q = 0$, indicates the case of homoscedastic error variance.

One of the shortcomings of an ARCH(q) model is that there are q+1 parameters to estimate. The accuracy of model estimation might be lost as q becomes a large number. This same issue was investigated in the discussion of parsimony in ARMA modeling in section 3.

The generalized ARCH model, or GARCH, is an alternative method for capturing long-lagged effects with fewer parameters. First, consider equation (25) and rewrite it as[9)]

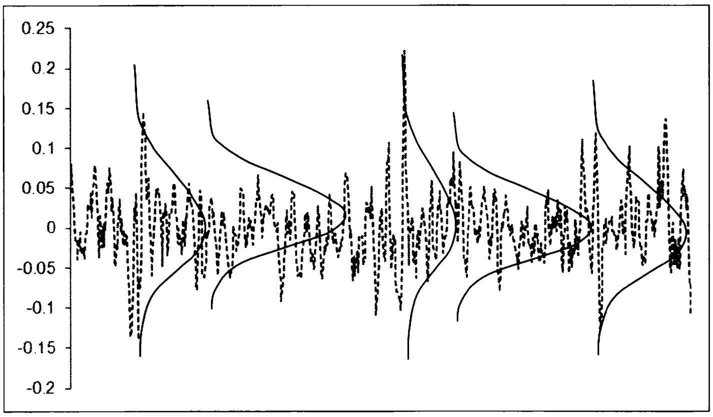
$$h_t = \sigma_t^2 = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \beta_1\alpha_1\varepsilon_{t-2}^2 + \beta_1^2\alpha_1\varepsilon_{t-3}^2 + \cdots \tag{26}$$

where α_1, α_2 , $\cdots$, $\alpha_s = \alpha_1\beta_1^{1-1}$, $\alpha_1\beta_1^{2-1}$, $\cdots$, $\alpha_1\beta_1^{s-1}$. Next, subtract and add $\beta_1\alpha_0$ in equation (26),

8) Engle [2001a] explains the updating rule and an intuition underlying the ARCH model by using a hypothesized numerical example.

9) See Griffith, Hill and Lim [2008], p.372.

Figure 5: Time Varying Volatility



Source: Alexander [2001], p.13.

$$h_t = \alpha_0 - \beta_1\alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \beta_1\alpha_0 + \beta_1\alpha_1\varepsilon_{t-2}^2 + \beta_1^2\alpha_1\varepsilon_{t-3}^2 + \cdots$$

$$= (\alpha_0 - \beta_1\alpha_0) + \alpha_1\varepsilon_{t-1}^2 + \beta_1(\alpha_0 + \alpha_1\varepsilon_{t-2}^2 + \beta_1\alpha_1\varepsilon_{t-3}^2 + \cdots) \quad (27)$$

According to equation (27), the variance of ε at time $t-1$ is expressed as

$$h_{t-1} = \alpha_0 + \alpha_1\varepsilon_{t-2}^2 + \beta_1\alpha_1\varepsilon_{t-3}^2 + \beta_1^2\alpha_1\varepsilon_{t-4}^2 + \cdots \quad (28)$$

Then, equation (24) can be rewritten as

$$h_t = (\alpha_0 - \beta_1\alpha_0) + \alpha_1\varepsilon_{t-1}^2 + \beta_1 h_{t-1}$$

$$h_t = \sigma_t^2 = \omega + \alpha_1\varepsilon_{t-1}^2 + \beta_1 h_{t-1} \quad (29)$$

where $\omega = (\alpha_0 - \beta_1\alpha_0)$. This is the GARCH(1,1) model. σ_t^2 is the conditional variance.

The GARCH(1,1) model states that the current fitted variance, h_t is interpreted as a weighted function of a long-term average value (dependent on ω) and information about volatility during the previous period ($\alpha_1\varepsilon_{t-1}^2$) and fitted variance from the model during the previous period ($\beta_1 h_{t-1}$). Large coefficient α_1 means that volatility reacts quite intensely to market movements of the previous period (i.e. the ARCH term is a reaction coefficient). Large coefficient β_1 indicates that shocks to conditional variance in the previous period are persistent and take a long time to die out, so volatility is persistent (i.e. the GARCH term is a persistence coefficient). If α_1 is relatively high and β_1 is relatively low then volatility tend to be more 'spiky' (large reaction and low persistence). The sum of $\alpha + \beta$ is referred to as the persistence of the conditional variance process. The positivity of h_t is ensured by the restrictions: $\omega > 0.\ \alpha \geq 0$, and $\beta \geq 0$. This GARCH(1,1) model is a special case of the more general GARCH(p,q) model, where p is the number of lagged h terms and q is the number of lagged ε^2 terms. It is worth noting that GARCH(p,q) modeling of the conditional variance is analogous to ARMA(p,q) modeling of the conditional mean.

The GJR model: asymmetric GARCH model

Positive and negative news are treated asymmetrically in the financial markets. It has been argued that negative news about stock returns is likely to cause volatility to rise by more than positive news of the same magnitudes. Such asymmetries are often called leverage effects (Figure 6). The first volatility cluster illustrates that there is turbulence in the financial market following an unexpected piece of bad news and the second one indicates an expected announcement of good news.

The threshold ARCH model (i.e. T-ARCH) is a simple extension of GARCH with an additional term added to account for possible asymmetries. The T-GARCH model is also referred to the GJR model, named after the authors Glosten, Jagannathan and Runkle [1993]. In the GJR version of the model, the specification of the conditional variance is:

$$h_t = \omega + \alpha_1 \varepsilon_{t-1}^2 + \gamma d_{t-1} \varepsilon_{t-1}^2 + \beta_1 h_{t-1} \tag{30}$$

$$d_t = \begin{cases} 1 & \varepsilon_t < 0 \ (bad\ news) \\ 0 & \varepsilon_t \geq 0 \ (good\ news \end{cases} \tag{31}$$

where γ is known as the asymmetry or leverage term. When γ is 0, the GJR model converges to the standard GARCH form. On the other hand, when the shock is positive (i.e. good news) the effect on volatility is α_1 but when the news is negative (i.e. bad news) the effect on volatility is $\alpha_1+\gamma$. Thus, so long as γ is significant and positive, negative shocks have a larger effect on h_t than positive shocks.

The ARMA-GARCH & GJR results are given in Table 4. The mean equations used here are specified by ARMA modeling in Section 3. The main output of parameter estimation is divided into two parts: the mean equation in the upper part and the variance equation in the lower part. The conditional mean equation is specified in an ARMA(p,q) model, whose function consists of the following terms:

- a constant term: μ
- autoregressive (AR) terms: $\emptyset_p$
- moving average (MA) terms: θ_q.

The conditional variance equation is specified in an GARCH(1,1) model, whose function consists of the three terms:

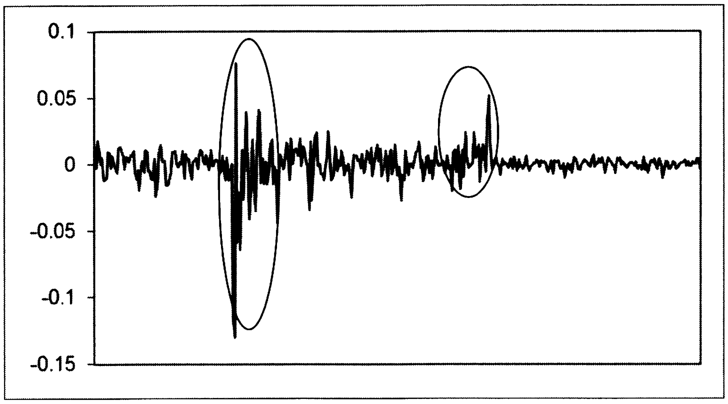
- a constant term: ω
- the ARCH term: α_1

which means a reaction coefficient to news about volatility from the previous period, measured as the lag of the squared residual from the mean equation: ε_{t-1}^2 .

- the GARCH term: β_1

which means a persistence coefficient to last period's forecast variance: $\sigma_{t-1}^2 = h_{t-1}$.

Figure 6: Leverage Effects



Source: Alexander [2001], p.13.

Engle [2001a] interprets this specification as an updating rule of adaptive or learning behavior of an agent or trader in a financial context. The trader predicts this period's variance h_t by forming a weighted average of a long term average (the constant, ω), the forecast variance from last period (the GARCH term, β_1) and information about volatility observed in the previous period (the ARCH term, α_1). The trader must estimate ω, α, β; updating simply requires knowing the previous forecast h_{t-1} and residual ε_{t-1}. The weights of the updating rule are $(1-\alpha_1-\beta_1, \alpha_1, \beta_1)$. Under the restriction that the weights are positive, requiring $\alpha_1 > 0, \beta_1 > 0, \omega > 0,$ this only works if $\alpha_1 + \beta_1 < 1$. If the asset return observed today was unexpectedly highly volatile, then the trader will increase the estimate of the variance for the next period. The conditional variance equation is consistent with the phenomena of the volatility clustering, that is, the amplitude of the return varies over time.

Table 4 shows that the parameter restrictions are fulfilled for all hedge fund indices. The coefficients on both the lagged squared residual and lagged conditional variance terms in the conditional variance equation are highly statistically significant for all hedge fund index returns. The persistence of the volatility is measured as the sum of $\hat{\alpha}$ and $\hat{\beta}$. The results indicate that the volatility of hedge fund returns is quite persistent. Especially, the sum of $\hat{\alpha}$ and $\hat{\beta}$ for Macro/CTA and Relative Value Arbitrage is very close to unity (approximately 0.99). This implies that shocks to the conditional variance will be highly persistent and a large positive and a large negative return will lead future forecasts of the variance to be high for a subsequent period. A volatility of half-life (i.e. the half-life period: HLP) takes 22.757 days for the Equity Hedge and 29.921 days for the Event Driven, whereby the HLP of 69.668 and 147.131 days for Macro/CTA and Relative

Table 4 : ARMA-GARCH & GJR Modeling

	ARMA-GARCH(1,1) modeling				ARMA-GJR(1,1) modeling			
	Equity Hedge	Event Driven	Macro/CTA	Relative Value Arbitrage	Equity Hedge	Event Driven	Macro/CTA	Relative Value Arbitrage
	AR(1)	ARMA(1,2)	AR(1)	ARMA(1,2)	AR(1)	ARMA(1,2)	AR(1)	ARMA(1,2)
Mean equation								
$\hat{\mu}$	0.0288***	0.0319***	-0.0007	0.0239***	0.0162	0.0275***	0.0051	0.0156*
	(0.0071)	(0.0055)	(0.0067)	(0.0058)	(0.0076)	(0.0054)	(0.0064)	(0.0081)
$\hat{\Phi}_1$	0.1791***	0.3063	0.0699***	0.9574***	0.1878***	0.3809*	0.0551***	0.9715***
	(0.0198)	(0.2503)	(0.0209)	(0.0131)	(0.0197)	(0.2210)	(0.0200)	(0.0106)
$\hat{\theta}_1$	—	-0.2034	—	-0.9009***	—	-0.2745		-0.9109***
		(0.2510)		(0.0249)		(0.2219)		(0.0246)
$\hat{\theta}_2$	—	0.0390	—	-0.0107	—	0.0337		-0.0080
		(0.0366)		(0.0221)		(0.0351)		(0.0224)
Variance equation								
$\hat{\omega}$	0.0045***	0.0018***	0.0020***	0.0006***	0.0077***	0.0027***	0.0011**	0.0006***
	(0.0010)	(0.0004)	(0.0005)	(0.0002)	(0.0014)	(0.0006)	(0.0005)	(0.0002)
$\hat{\alpha}_1$	0.1080***	0.0998***	0.0851***	0.1241***	0.0119	0.0448***	0.0961***	0.0737***
	(0.0187)	(0.0160)	(0.0112)	(0.0224)	(0.0193)	(0.0168)	(0.0147)	(0.0263)
$\hat{\gamma}$	—	—	—	—	0.1723***	0.0925***	-0.0666***	0.0861**
					(0.0284)	(0.0263)	(0.0160)	(0.0421)
$\hat{\alpha}+\hat{\gamma}$	—	—	—	—	0.1842	0.1373	0.0295	0.1598
$\hat{\beta}_1$	0.8620***	0.8773***	0.9051***	0.8712***	0.8392***	0.8668***	0.9344***	0.8778***
	(0.0183)	(0.0151)	(0.0122)	(0.0187)	(0.0182)	(0.0162)	(0.0123)	(0.0153)
$\hat{\alpha}_1+\hat{\beta}_1$	0.9700	0.9771	0.9901	0.9953				
HLP	22.757	29.921	69.668	147.131				
SIC	0.7472	0.0422	0.7750	-0.5957	0.7266	0.0367	0.7672	-0.6023
LogL	-1049.69	-32.5273	-1089.5	880.6709	-1016.22	-20.6415	-1074.31	894.0463
ARCH effect:								
ARCH LM(1) test	1.9547	0.7942	0.0003	0.4168	3.9839**	1.1978	2.3625	1.4231
Standarized Residuals:								
Mean	-0.0429	-0.0271	0.0178	-0.0275	-0.0126	-0.0105	0.0000	-0.0051
Std. Dev.	0.9985	0.9993	0.9996	0.9990	0.9995	0.9998	0.9997	0.9993
Skewness	-0.4978	-0.4286	-0.5018	-0.1246	-0.4515	-0.4309	-0.4624	-0.0361
Kurtosis	4.8586	5.1456	6.5193	5.8927	4.9037	5.2338	5.8252	6.1179
Jarque-Bera	530.33***	636.844***	1597.628***	1005.583***	529.625***	683.873***	1054.157***	1160.280***
Ljung-Box statistic *Ho: no-autocorrelation*								
: Q(12)	6.968	20.180**	3.754	12.672	7.144	19.107**	4.004	8.516
: Q(12)	19.385**	16.435*	3.118	16.047*	15.456	16.182**	11.550	14.330

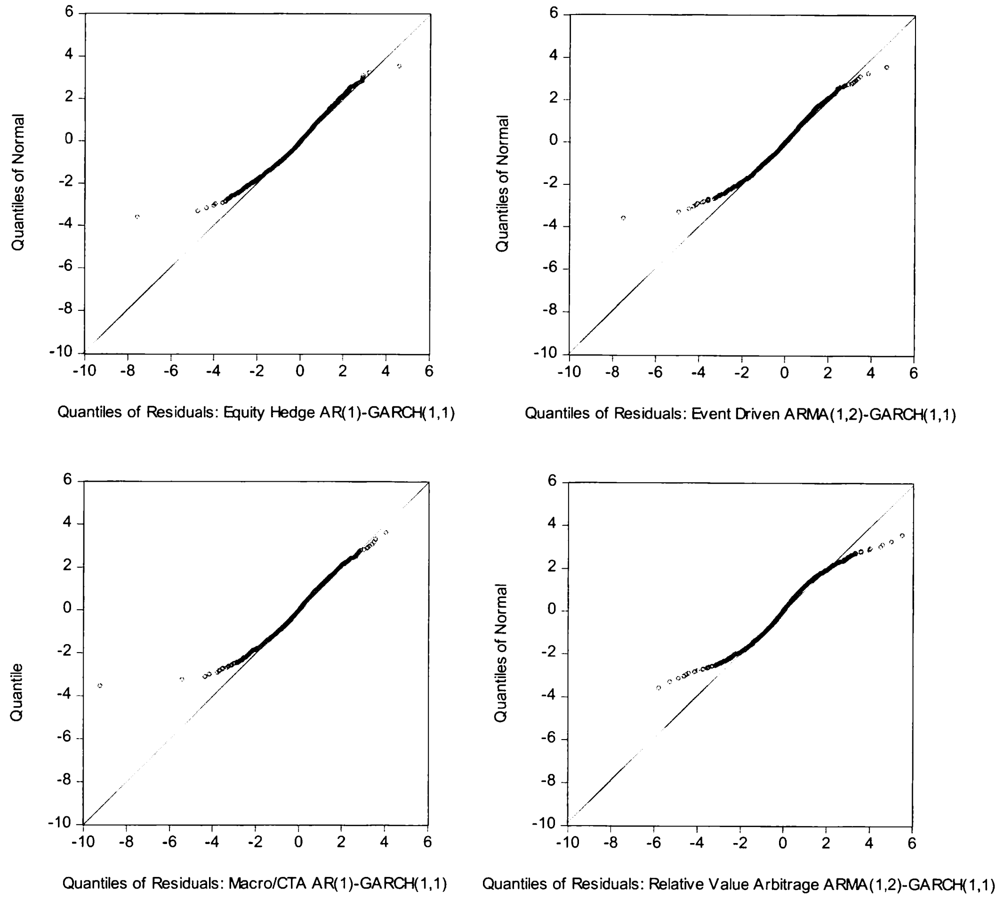
Notes: Based on daily, -continuously compounded returns for 2864 observations 04/01/2003 to 08/11/2014; standard errors are presented in parenthesis; The statistical significance is determined by using Bollerslev-Wooldridge robust standard errors; ***, **, * denote significance at 99%, 95% and 90% confidence levels.

Value Arbitrage are much higher.[10] Therefore, the return volatilities of four hedge fund indices have quite long memories. In addition, the sum of $\hat{\alpha}$ and $\hat{\beta}$ is significantly less than one, which implies the volatility process does return to its mean (Engle and Patton [2001], p.16), so-called mean reverting behavior.

The ARCH LM(1) test determines whether there are any remaining ARCH effects in the residuals. The

10) Füss, Kaiser and Adams [2006] compute the length until half of the volatility (i.e. the half-life period: HLP) as $\mathrm{HLP} = \log(0.5)/\log(\hat{\alpha}+\hat{\beta})$.

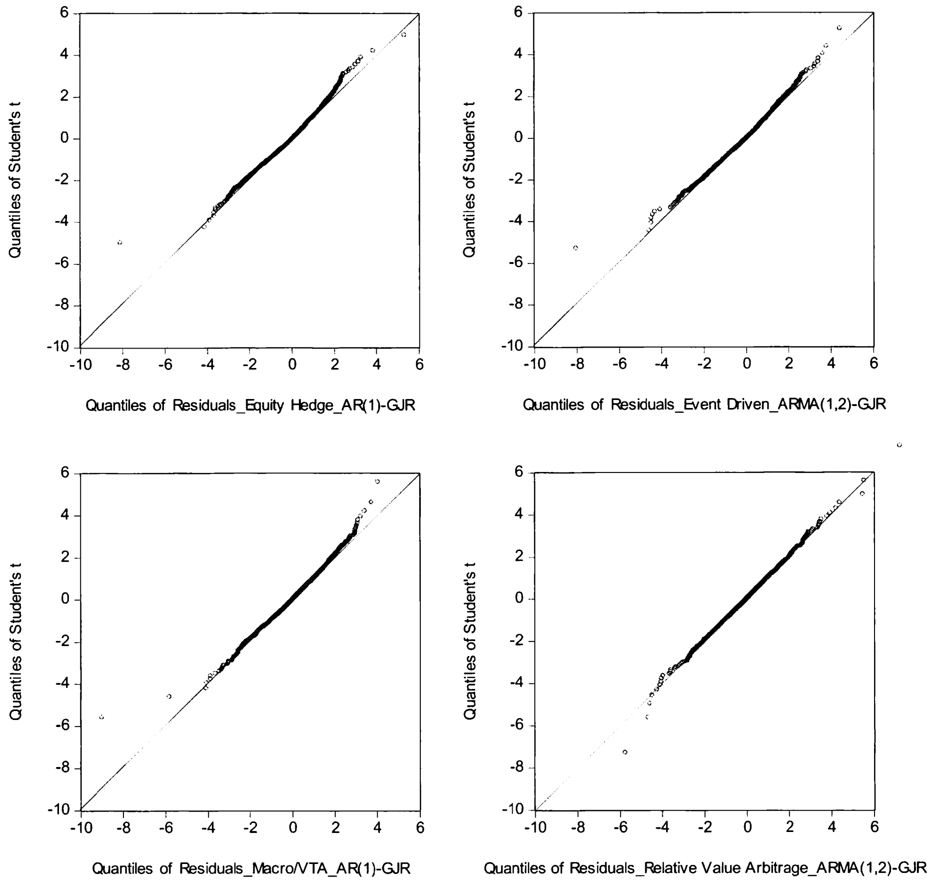
Figure 7: The Standardized Residuals of the GARCH Estimation Against the QQ-Plot of the Normal Distribution



null hypothesis is that no ARCH effect in the residuals is not rejected for all hedge fund indices in ARMA-GARCH(1,1) modeling. The ARCH-LM(1) tests confirm the null hypothesis of no first order ARCH effects in the squared residuals of the models for four hedge fund index return-series. This result means that the ARMA-GARCH modeling takes the heteroscedasticity and the changing unconditional and conditional variance in the return-series into account. Finally, the Jarque-Berra statistics of ARMA-GARCH estimation suggests that skewness and kurotosis in the standardized residuals are reduced from the ones of ARMA estimation (Table 3) but not completely eliminated.

One way of further testing the distribution of the residuals is to plot the quantiles. If the residuals are normally distributed, the points in the QQ-plots should lie alongside a straight line. Figure 7 displays the QQ-plots for four index returns of the ARMA-GARCH modeling. The plots indicate that Equity Hedge and Macro/CTA are primarily large negative shocks that are driving the departure from normality and Event

Figure 8: The Standardized Residuals of the GJR Estimation Against the QQ-Plot of the Student-t Distribution



Driven and Relative Value Arbitrage are primarily large negative shocks and are also relatively positive shocks that are driving the departure from normality.

Next, an examination of asymmetric effects on the conditional variance is conducted through assessment of ARMA-GJR modeling. The coefficient α implies an impact of good news, while the sum of the $\hat{\alpha}+\hat{\gamma}$ implies an impact of bad news. The coefficient $\hat{\gamma}$ from Table 4 is positive for Equity Hedge, Event Driven, and Relative Value Arbitrage, and statistically significant. There is the largest leverage effect for Equity Hedge since the coefficient $\hat{\gamma}$ is 0.1723. However, the coefficient $\hat{\gamma}$ is negative for Macro/CTA, provided that $\hat{\alpha}+\hat{\gamma}$ is 0.0295 ≥ 0. The specification of the GJR model is still admissible. All hedge fund index return series seem to prefer the GJR model to the GARCH model since all values of SIC decrease and ones of log likelihood function increase in the ARMA-GJR(1,1) modeling from the ARMA-GARCH(1,1) modeling. Finally, the standardized residuals are independently identically distributed if the selected model is the true model. It can be checked by the Ljung-Box statistics, LB-Q(12) for testing on dependencies of the standardized residuals and the standardized residuals squared. For Equity Hedge, Macro/CTA, and Relative Value Arbitrage no-dependencies of the standardized residuals are confirmed and the standardized residuals are squared. Only for Event Driven the null hypothesis of no-autocorrelations rejected. Finally, it is important to note that the descriptive statistics of the standardized residuals for all hedge fund index returns exhibit negative skewness and leptokurtosis (i.e. excess kurtosis), which means that the distributions of the standardized residuals are fat tailed distributions. To model the thick tail in the residuals, the standardized residuals are assumed to follow a Student-t distribution. Figure 8 illustrates that the large negative and positive residuals more closely follow a straight line than those of Figure 7. The residual distributions were close to Student-t distributions, that is, the fat-tailed distributions.

5. Concluding Remarks

In this paper, the linear ARMA type models and the non-linear volatility models such as GARCH(1,1) and GJR(1,1) for four primary hedge fund strategies were estimated and compared with each other. Through the ARMA modeling, the estimated ARMA processes for the returns to the non-directional strategies such as Event Driven and Relative Value Arbitrage exhibit are highly serially correlated, on the other hand, the directional strategies such as Equity Hedge and Macro/CTA show relatively low serial correlation.

The return series of four hedge fund indices depict volatility clustering, that is, time varying volatility. To capture the autoregressive conditional heteroscedasticity, the GARCH structure for hedge fund index returns, in addition to the ARMA structure, are specified. The examination of the ARMA-GARCH modeling of hedge fund strategy returns shows the asymmetric effect to volatility, and thus, the GJR models are selected although their conditional volatilities shows significant differences in persistence and the direction of asymmetry. Finally, it is worth noting that the distributions of the standardized residuals for all hedge fund strategies reveal leptokurtosis and the residuals against the quantiles of the Student-t distribution more closely follow a straight line than those of the normal distribution. Volatility models, such as the GARCH type approach are often applied to Value at Risk. In the case of VaR measurement including time-varying conditional volatility, it is important to recognize that the residuals distribution follows the fatter tailed distribution than the normal one for downside risk evaluation.

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