A game theoretical analysis of FTA network formation

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Abstract

We would like to analyze a free trade agreement (FTA) network formation or collapse processes and the stable network.

For the purpose, we consider a 3-stage game. Each country has a government, a firm, and a market. At first, every government decides to make FTA each other. Second, every government decides the tariff rate each other (The rate is zero if the governments made FTA.). Third, each firm faces Cournot oligopoly on every market and decides the amount of the export and domestic products.

For simplicity, we investigate the case $n = 3$. As a result, one FTA and three FTA (the complete network) are the stable networks.

Keywords: FTA, Network Formation, Cournot competition

JEL classification: C70; F10; D21

1 Introduction

We consider an FTA formation model which is described as a 3-stage game. Each country has a government, a firm, and a market. At first, every government decides to make FTA each other. Second, every government decides the tariff rate each other. If the firms are connected by an FTA, they can make free trade. Otherwise, they are induced the strategic tariff Third, given the governments' trade policies, each firm faces Cournot oligopoly in every market and decides the amount of the export and domestic products.

We assume that each firm maximizes the profit and that each government maximizes the total amount of the firm's profit, consumers' surplus, and the tariff revenue of the country. We can clarify the formation (or collapse) process of the FTA network by using backward induction and the definition of stable network by Jackson and Wolinsky (1996) .

In the literature, the formation of a network is studied by Jackson and Wolinsky (1996), Dutta and Mutuswami (1997), Watts (2001), Jackson and Watts (2002) and so on. They analyze the endogenous network formation, the stable network, and the reasonable allocation in general settings. Goyal and Joshi (2003) apply the model to oligopoly cases. Kawamata and Tamada (2004) and Hirase (2012) especially pay attention to Cournot competition. Bilateral FTA is introduced by Goyal and Joshi (2006). FTA network formation in the differentiated industrial commodities case is studied by Furusawa and Konishi (2007). In our model, we focus the formation or collapse process of the FTA network of the homogenous good using partial equilibrium analysis. We discuss also the incentives of the outsider of the FTA.

The rest of the paper is composed as follows. The general terms and notations are defined in Section 2. The example of the three country case is shown in Section 3. Section 4 is the concluding remarks.

2 The Model

2.1 Market

Let $N = \{1, 2, ..., n\}$ be a set of countries. Each country has one government, one firm and one market. Each firm produces a homogeneous good for the domestic market and the exports. In each market j, the firms faces a linear inverse demand function as follows:

$$p_j = a - Q_j, \quad a >> 0, \tag{1}$$

where p_j is the price and Q_j is the amount of the demand. And they play a Cournot competition.

The marginal cost of each firm i is given by c. For convenience, we also assume $a > c > 0$ and denote the amount of firm i's product for market j as q_{ij}.

Before the competition, each government of the country decides to make FTA or not, and then induces the strategic tariff on the firm which is not linked to that country. We denote the tariff rate of government i to firm j as t_{ij}, which implies that $t_{kl} = 0$ if the country k and j makes FTA. We assume $t_{ii} = 0$ for any country $i \in N$. Thus, we can consider a 3-stage game. At 1st stage, FTA network forms. At 2nd stage, each government given a network decides the tariff levels. At 3rd stage, each firm decides the amount of the products.

Given the tariff levels and the amounts of the products firm i's profit from market j. π_{ij} is as follows.

$$\pi_{ij} = p_j q_{ij} - c q_{ij} - t_{ji} q_{ij} = (a - Q_j - c - t_{ji}) q_{ij} = (a - \sum_{k \in N} q_{kj} - c - t_{ji}) q_{ij} \qquad (\forall i, j \in N). \quad (2)$$

The profit of firm i, π_i is defined as $\pi_i = \sum_{j \in N} \pi_{ij}$. The consumers surplus at market i is defined as usual:

$$CS_i := \frac{1}{2} Q_i \qquad (\forall i \in N). \quad (3)$$

Government i's tariff revenue is

$$T_i := \sum_{j \in N} t_{ij} q_{ji} \qquad (\forall i \in N). \quad (4)$$

where $t_{kj} = 0$ if country k and j make FTA and $t_{ii} = 0$. The welfare of country i is defined as follows:

$$W_i = CS_i + \pi_i + T_i \qquad (\forall i \in N). \quad (5)$$

By backward induction, each government decides the tariff level to maximize the country's welfare. For convenience, we assume that government i induces no tariff to FTA countries and a constant tariff level t_i to the other countries. That is, $t_{ij} = t_{ik} = t_i$ if country i have not made FTA with country j and country k, and $t_{il} = 0$ if country i and country l have made FTA.

2.2 FTA Network

The terms on network are same as Hirase (2012). A network g on N is a set of pairs of the members of N, $g \subset \{\{i, , j\} | i, j \in N, i \neq j\}$. An element of g is called a link. $\{i, j\}$ is the link between i and j which means a FTA between i and j.

We focus on the case three countries in the next section. The network patterns of the case is summarized Figure 1.

2.3 Stability

In the game, each pair of the countries decides to form or sever a link and coalition structure is formed at first stage. And then, the firms play the Cournot competition as second stage. In this subsection, we define the stability of the network.

For convenience, let $\pi_i(g)$ be the profit of firm i for a given network g. The firms have the opportunity to form new links or sever existing links for a network g. According to Jackson and Wolinsky (1996), we define the pairwise stable network as follows:

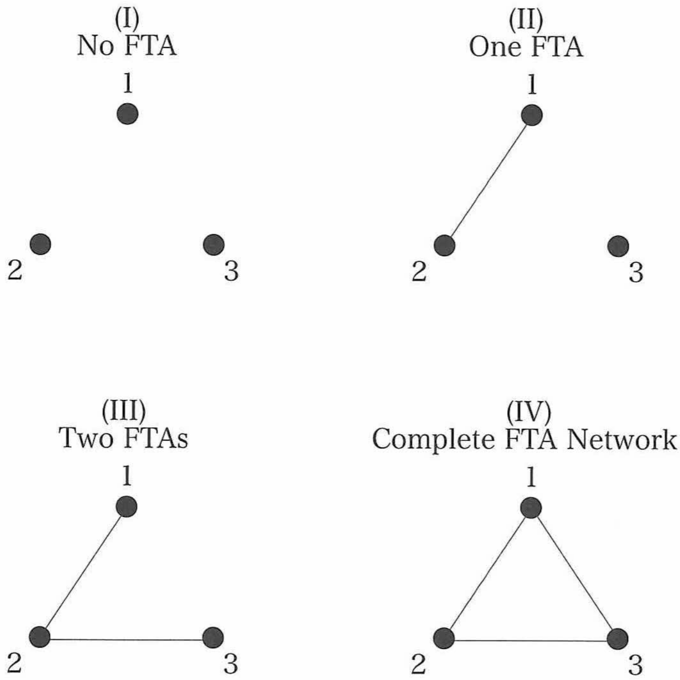

Figure 1: networks ($n = 3$)

Definition 1. A network g is *pairwise stable* if two conditions below are satisfied.

1. $\forall \{i, j\} \in g, \pi_i(g) > \pi_i(g \setminus \{i, j\})$ and $\pi_j(g) > \pi_j(g \setminus \{i, j\})$.

2. $\forall \{i, j\} \notin g$, if $\pi_i(g) < \pi_i(g \cup \{i, j\})$, then $\pi_j(g) > \pi_j(g \cup \{i, j\})$.

2.4 Equilibrium

From a given network g, we can derive the best response function of firm i by first order condition.

$$q_{ij} = \frac{a - c - \sum_{k \neq i} q_{kj} - t_{ji}}{2} \qquad (\forall i, j, k \in N). \tag{6}$$

At an equilibrium, each equation above is satisfied. Hence, from the assumption, the production amount at an equilibrium is as follows (* means the equilibrium).

$$q_{ii}^* = \frac{a - c + (n+1)t_{t_i}}{n+1} \qquad (\forall i \in N) \qquad \text{and} \qquad q_{ij}^* = \frac{a - c - 2t_j}{n+1} \qquad (\forall i \neq j). \tag{7}$$

The tariff levels at the equilibrium satisfy all equations (first order conditions) bellow.

$$\frac{\partial CS_i}{\partial t_i} = 0, \frac{\partial \pi_{ii}}{\partial t_i} = 0, \text{ and } \frac{\partial T_i}{\partial t_i} = 0 \qquad (\forall i \in N). \tag{8}$$

We investigate the solutions of the three country case and interpret it in the next section.

3 Example

Suppose that n=3. Without loss of generality, we can obtain four FTA situations shown in Figure 1(I, II, III, and IV). The solutions of the four situations are described in the four subsections below.

3.1 No FTA (I)

3.1.1 3rd stage

$$q_{ii} = \frac{a-c+2t_i}{4} \quad (\forall i \in N) \quad \text{and} \quad q_{ij} = \frac{a-c-2t_i}{4} \quad (\forall i \neq j).$$

3.1.2 3.1.2. 2nd stage

$$\begin{aligned} CS_i &= \frac{1}{2}(\frac{3(a-c)-2t_i}{4})^2 \qquad (\forall i \in N), \\ \pi_{ii} &= (\frac{a-c+2t_i}{4})^2 \qquad (\forall i \in N), \\ \pi_{ij} &= \frac{(a-c)^2-4t^2}{16} - \frac{a-c+2t_i}{4}t_i \qquad (\forall i \neq j), \text{ and} \\ T_i &= t_i\frac{a-c-2t_i}{2} \qquad (\forall i \in N). \end{aligned}$$

Hence,

$$t_i = \frac{3(a-c)}{10} \quad (\forall i \in N), \quad q_{ii} = \frac{2(a-c)}{5} \quad (\forall i \in N), \quad \text{and} \quad q_{ij} = \frac{(a-c)}{10} \quad (\forall i \neq j).$$

3.1.3 Welfare

$$\begin{aligned} CS_i &= \frac{9(a-c)^2}{50} \qquad (\forall i \in N), \\ \pi_{ii} &= \frac{4(a-c)^2}{25} \qquad (\forall i \in N), \\ \pi_{ii} &= \frac{(a-c)^2}{100} \qquad (\forall i \neq j), \text{ and} \\ T_i &= \frac{3(a-c)^2}{50}. \end{aligned}$$

Hence,

$$W_i = \frac{21(a-c)^2}{50} = 0.42(a-c)^2 \qquad (\forall i \in N). \tag{9}$$

3.2 One FTA (II)

3.2.1 3rd stage

$$\begin{aligned}
q_{11} &= q_{22} = q_{12} = q_{21} = \frac{a-c+t_1}{4}, \\
q_{13} &= q_{23} = \frac{a-c-2t_3}{4}, \\
q_{31} &= q_{32} = \frac{a-c-3t_1}{4}, \\
q_{33} &= \frac{a-c+2t_3}{2}. \text{ and} \\
t_1 &= t_2.
\end{aligned}$$

3.2.2 2nd stage

$$\begin{aligned}
t_1 &= t_2 = \frac{(a-c)}{7}, \\
t_3 &= \frac{(a-c)}{3}, \\
q_{11} &= q_{12} = q_{21} = q_{22} = \frac{2(a-c)}{7}, \\
q_{13} &= \frac{(a-c)}{12}, \\
q_{31} &= q_{32} = \frac{(a-c)}{7}, \text{ and} \\
q_{33} &= \frac{(5a-c)}{12}.
\end{aligned}$$

3.2.3 Welfare

$$\begin{aligned}
CS_1 &= CS_2 = \frac{25(a-c)^2}{98}, \\
CS_3 &= \frac{49(a-c)^2}{288}, \\
\pi_{11} &= \pi_{22} = \frac{4(a-c)^2}{49}, \\
\pi_{13} &= \pi_{23} = \frac{(a-c)^2}{144}, \\
\pi_{31} &= \pi_{32} = \frac{(a-c)^2}{49}, \\
\pi_{33} &= = \frac{25(a-c)^2}{144}, \\
T_1 &= T_2 = \frac{(a-c)^2}{49}, \text{ and} \\
T_3 &= \frac{(a-c)^2}{18}.
\end{aligned}$$

Hence,

$$W_1 = W_2 = \frac{3145(a-c)^2}{7056} \simeq 0.446(a-c)^2 \quad \text{and} \quad W_3 == \frac{6211(a-c)^2}{14112} \simeq 0.440(a-c)^2. \quad (10)$$

3.3 Two FTAs (III)

3.3.1 3rd stage

$$\begin{aligned}
q_{11} &= q_{33} = q_{21} = q_{23} = \frac{a-c+t_1}{4}, \\
q_{12} &= q_{32} = \frac{a-c}{5}, \\
q_{13} &= q_{31} = \frac{a-c-3t_1}{4}, \\
q_{22} &= \frac{3(a-c)}{10}, \text{ and} \\
t_1 &= t_3.
\end{aligned}$$

3.3.2 2nd stage

$$\begin{aligned}
t_1 &= t_3 = \frac{(a-c)}{7}, \\
q_{11} &= q_{33} = q_{21} = q_{23} = \frac{2(a-c)}{7}, \text{ and} \\
q_{13} &= q_{31} = \frac{a-c-}{7}.
\end{aligned}$$

3.3.3 Welfare

$$
\begin{aligned}
CS_1 &= CS_3 = \frac{25(a-c)^2}{98}, \\
CS_2 &= \frac{49(a-c)^2}{200}, \\
\pi_{11} &= \pi_{33} = \frac{4(a-c)^2}{49}, \\
\pi_{12} &= \pi_{32} = \frac{3(a-c)^2}{50}, \\
\pi_{13} &= \pi_{31} = \frac{2(a-c)^2}{49}, \\
\pi_{21} &= \pi_{23} = \frac{4(a-c)^2}{49}, \\
\pi_{22} &= = \frac{9(a-c)^2}{100}, \\
T_1 &= T_3 = \frac{(a-c)^2}{49}, \text{ and} \\
T_2 &= 0.
\end{aligned}
$$

Hence,

$$
W_1 = W_3 = \frac{512(a-c)^2}{1225} \simeq 0.418(a-c)^2, \qquad \text{and} \qquad W_2 = \frac{6211(a-c)^2}{14112} \simeq 0.498(a-c)^2. \tag{11}
$$

3.4 Complete FTA network (IV)

3.4.1 3rd stage

$$
q_{ij} = \frac{a-c}{4} \qquad (\forall i, j \in N).
$$

3.4.2 2nd stage

Each tariff level is zero because of the Complete FTA network.

3.4.3 Welfare

$$
\begin{aligned}
CS_i &= \frac{9(a-c)^2}{32} \qquad (\forall i \in N), \\
\pi_{ii} &= \frac{(a-c)^2}{16} \qquad (\forall i \in N), \\
\pi_{ii} &= \frac{(a-c)^2}{16} \qquad (\forall i \neq j), \text{ and} \\
T_i &= 0.
\end{aligned}
$$

Hence,

$$W_i = \frac{15(a-c)^2}{32} \simeq 0.469(a-c)^2 \qquad (\forall i \in N). \tag{12}$$

4 Concluding Remarks

The result of the example in Section 3 is summarized in Figure. 2.

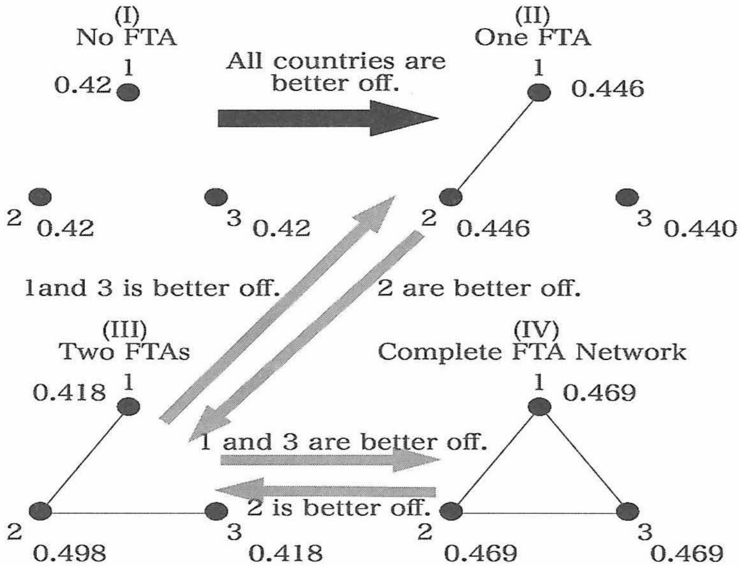

Figure 2: network formation

- From the definition, both one FTA (I) and the complete FTA network (IV) are stable networks.
- (I)-(II): If no FTA exists, any single FTA is supported by all countries (including the outsider). This means that the FTA between country 1 and 2 is supported by not only themselves but also country 3 which is outside of the free trade.
- (II)-(III): The country which has made one FTA has an incentive to make the FTA with country which has made no FTA. On the other hand, the country which has made no FTA does not have incentive to make FTA with the country which has made one FTA. From the definition of the stability, FTA between 1 and 3 is not made.

- (III)-(IV): At (III), both country 1 and country 3 have incentive to make FTA. However, country 2 would like to remain the hub (center) position of the trades. It has an incentive to interfere the FTA formation between country 1 and 3. This *outsider's motivation* has implications for the multi-country FTA formations or negotiations.
- Any country would like to be the hub or center position of the trade (have many structural holes). However, the star or line network (two FTAs (III)) can not be stable.
- Future problem: It is to generalize the investigation and the model. We can investigate n country case. We can weaken one homogenous (good and country) settings and the assumptions on the demand function. And we would like to use the general equilibrium analysis.

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