Joint Projects with Commitments to the Final Step

Yusuke Samejima

Abstract

We investigate a variant of the two-player voluntary contribution game studied by Compte and Jehiel [2003]. Compte and Jehiel assume that alternate contributions for completing a joint project are sunk costs and agents cannot commit in advance to a specific sequence of contributions, as is also assumed in Admati and Perry [1991]. We slightly change their assumption as follows: agents can commit to proposals such as: "When it comes to a situation where my final payment of a certain amount completes the project, I will pay the amount then for sure." Although such a commitment to the final step seems a generous proposal at a glance, the commitment gives the proposer a great advantage in his equilibrium payoff.

1. Introduction

This paper investigates a two-player voluntary contribution game in which agents can commit in advance to make a certain amount of payment at the final step of completing a joint project before the agents start alternate contributions that become sunk costs. The game might be regarded as a combination of the contribution game and the subscription game mentioned in Admati and Perry [1991]. They explain the difference between the contribution game and the subscription game as follows. In the contribution game, commitments and enforceable contracts are not available, and the cost of contributions is sunk. In the subscription game, agents can make conditional commitments to contribute in the future, and the cost of contributions is borne only when enough contributions are pledged to complete the project.

In their analysis of the contribution game, Admati and Perry's main concern is a pattern of contributions. They show that contributions are made in small steps along the equilibrium path. They suggest that the sunk character of contributions is a source of such a step-by-step pattern of contributions. However, Compte and Jehiel [2003] point out that Admati and Perry's result depends on convexity of a cost function and symmetry of agents' valuations of the project. Compte and Jehiel introduce a linear cost function and asymmetric valuations into Admati and Perry's contribution game, and show that at most two large contributions are realized in equilibrium.

Aside from the pattern of contributions, the present paper concerns how agents split the social surplus in equilibrium. In the equilibrium of Compte and Jehiel's game, the agent with the lower valuation gets all the surplus generated by completion of the project while the agent with the higher valuation gets a payoff of zero, which seems an unfair way of splitting the surplus. In the present paper, we introduce a different contribution protocol into Compte and Jehiel's model, and investigate how the equilibrium payoff profile changes. Specifically, our game gives agents options to make a proposal such as: "When it comes to a situation where my final payment of a certain amount completes the project, I will pay the amount then for sure." After making such proposals, the agents start an alternate contribution game. We assume that the agents can commit to the proposals at the final step.

In equilibrium of our game, agents split the social surplus in the following way. When only one agent has an option to propose, the proposer gets all the surplus while the other agent without the option gets a payoff of zero. On the other hand, when both agents can propose, the surplus is split relatively fairly between the two in the sense that the both agents get positive payoffs, although the second mover's payoff exceeds that of the first mover. Our result indicates that the commitment to the final step gives an agent a great advantage in his equilibrium payoff.

The remaining part of this paper is organized as follows. Section 2 explains our model of the two-player contribution game. Section 3 summarizes the previous results in the literature. Section 4 investigates the case where only one agent has an option to commit to the final step while Section 5 investigates the case where both agents have such options. Section 6 provides some concluding remarks.

2. The Model

We investigate a variant of the two-player voluntary contribution game studied by Compte and Jehiel [2003].

Two agents, agents 1 and 2, are the players of the game. They voluntarily contribute to complete a *project*, which costs $K > 0$. The project is a public good for the agents. On completion of the project, agent i obtains a benefit $V_i > 0$, which is called agent i's *valuation* of the project. Following Compte and Jehiel [2003], we focus on the case where $\max\{V_1, V_2\} < K < V_1 + V_2$, which means that neither agent can afford to complete the project alone and completion of the project is socially desirable.

The game is played in periods $t = 0, 1, 2, \ldots$. In period 0, each agent i can simultaneously make a *proposal* such as: "When it comes to a situation where my final payment of $C_i \geq 0$ at the end of period $t \geq 1$ completes the project, I will pay the amount to complete the project at the end of period t." We assume that the agents can commit to the proposals. At the end of period 0, (C_1, C_2) is observed by them.

From period 1, they contribute alternately as in the game studied by Compte and Jehiel [2003]. Agent 1 contributes in periods with positive odd numbers while agent 2 contributes in periods with positive even numbers until the project is completed. Let $m(t)$ denote the *mover* in period $t \geq 1$. That is, $m(t) = 1$ if t is a positive odd number while $m(t) = 2$ if t is a positive even number.

In the middle of period $t \geq 1$, agent i with $i = m(t)$ makes a *contribution* of an amount of $c_i^t \geq 0$. Since two agents take turns in making contributions, we require that $c_i^t = 0$ if $i \neq m(t)$: this constraint on (c_1^t, c_2^t) together with $c_1^0 = c_2^0 = 0$ for notational convenience is called the *feasibility* for contributions. We assume that contributions are non-refundable even if the project is not completed. So, contributions become sunk costs for the agents. At the end of period t, (c_1^t, c_2^t) is observed by them.

The *remaining amount* required for completion at the end of period t is denoted by

$$x^t = K - C_1 - C_2 - \sum_{\tau=0}^{t}(c_1^\tau + c_2^\tau).$$

If $x^t \leq 0$ at the end of period $t \geq 1$, then agents 1 and 2 fulfill their proposals by paying C_1 and C_2, respectively, at the end of period t.[1]

Let T denote the *period of completion* of the project, that is, T is the least natural number that satisfies the condition $x^T \leq 0$. When the project is completed, the game ends. If the project is not completed forever due to an insufficient amount of contributions, then we let $T = \infty$, and the game continues forever.

Let h^t denote a *history* at the beginning of period t: we define $h^0 = \emptyset$ and $h^t = \{(C_1, C_2), (c_1^0, c_2^0), \ldots, (c_1^{t-1}, c_2^{t-1})\}$ for $t \geq 1$. A history h^t is *non-terminal* if $x^{t-1} > 0$. A *terminal history* is denoted by h^{T+1}; it is an infinite sequence or a history with $x^T \leq 0$.

We focus on pure strategies. Agent i's *strategy* s_i is a function $s_i(h^t) \geq 0$ that associates with

[1] In our setting, even if $x^0 \leq 0$, agents fulfill their proposals at the end of period 1.

each non-terminal history h^t a non-negative real number. We interpret $s_i(h^0)$ as C_i while we interpret $s_i(h^t)$ as c_i^t for $t \geq 1$. By the feasibility for contributions, we require that $s_i(h^t) = 0$ if $i \neq m(t)$. A list of strategies (s_i, s_j) with $i \neq j$ is called a *strategy profile*.

Both agents discount benefits and contributions using a *discount factor* δ such that $0 < \delta < 1$. Agent i's *payoff* evaluated at the end of period 1 for a strategy profile (s_1, s_2) is given by

$$u_i(s_1, s_2) = \delta^{T-1}(V_i - C_i) - \sum_{t=1}^{T} \delta^{t-1} c_i^t$$

where C_i and c_i^t are variables that appear in the terminal history h^{T+1} realized by the strategy profile (s_1, s_2).

We look for subgame-perfect equilibria of the game. A strategy profile (s_1, s_2) is a *subgame-perfect equilibrium* of the game if, for every subgame of the game, the strategy profile induced by (s_1, s_2) is a Nash equilibrium of the subgame.[2]

Given a history $h^1 = \{(C_1, C_2), (c_1^0, c_2^0)\}$, let $\bar{V}_1 = V_1 - C_1$, $\bar{V}_2 = V_2 - C_2$, and $\bar{K} = K - C_1 - C_2$.[3] We denote a subgame that starts at a history h^1 at the beginning of period 1 by a list $(\bar{V}_1, \bar{V}_2, \bar{K})$.

3. The Previous Results

This section summarizes the results in the previous studies in the literature as *facts* in the context of our model. Throughout the section, we assume that the admissible strategies for each agent i are restricted in such a way that $C_i = 0$, i.e., each agent does not have an option to make a proposal to commit to the final step. We may regard that the previous studies assume $C_1 = C_2 = 0$ and analyze the subgame (V_1, V_2, K) starting in period 1 in our model. The facts discussed in this section apply to the subgame (V_1, V_2, K) as well as all its subgames starting at a non-terminal history.

Suppose that, for $t \geq 1$, we have a non-terminal history $h^t = \{(C_1, C_2), (c_1^0, c_2^0), \ldots, (c_1^{t-1}, c_2^{t-1})\}$.

Strategy $\bar{s}_i^*$ for agent i in the game such that $V_i \leq V_j$, $i \neq j$, and $C_1 = C_2 = 0$.

We fix $\bar{s}_i^*(h^0) = 0$ by the restriction $C_i = 0$. When $i \neq m(t)$, we require that $\bar{s}_i^*(h^t) = 0$ by the feasibility for contributions. When $i = m(t)$, the following descriptions define $\bar{s}_i^*(h^t)$.

(i) If $x^{t-1} \leq 0$, then let $\bar{s}_i^*(h^t) = 0$.

(ii) If $0 < x^{t-1} \leq (1-\delta)V_i$, then let $\bar{s}_i^*(h^t) = x^{t-1}$.

[2]For the formal definitions of subgame-perfect equilibria and Nash equilibria, readers are referred to Osborne and Rubinstein [1994].

[3]It is possible that $\bar{V}_1 \leq 0$, $\bar{V}_2 \leq 0$, or $\bar{K} \leq 0$.

(iii) If $(1-\delta)V_i < x^{t-1} \leq V_j$, then let $\bar{s}_i^*(h^t) = 0$.

(iv) If $V_j < x^{t-1} \leq \delta V_i + V_j$, then let $\bar{s}_i^*(h^t) = x^{t-1} - V_j$.

(v) If $\delta V_i + V_j < x^{t-1}$, then let $\bar{s}_i^*(h^t) = 0$.

Strategy $\bar{s}_j^{**}$ for agent j in the game such that $V_i \leq V_j$, $i \neq j$, and $C_1 = C_2 = 0$.

We fix $\bar{s}_j^{**}(h^0) = 0$ by the restriction $C_j = 0$. When $j \neq m(t)$, we require that $\bar{s}_j^{**}(h^t) = 0$ by the feasibility for contributions. When $j = m(t)$, the following descriptions define $\bar{s}_j^{**}(h^t)$.

(i) If $x^{t-1} \leq 0$, then let $\bar{s}_j^{**}(h^t) = 0$.

(ii) If $0 < x^{t-1} \leq V_j$, then let $\bar{s}_j^{**}(h^t) = x^{t-1}$.

(iii) If $V_j < x^{t-1}$, then let $\bar{s}_j^{**}(h^t) = 0$.

We first note that the following Fact 1 is essentially the same as Proposition C2 in Marx and Matthews [2000]; by applying their arguments in the proof of their proposition, the fact is obtained.

Fact 1. When $V_1 = V_2$, the strategy profiles $(\bar{s}_1^*, \bar{s}_2^{**})$ and $(\bar{s}_1^{**}, \bar{s}_2^*)$ are subgame-perfect equilibria of the game with the restrictions $C_1 = C_2 = 0$. Accordingly, the strategy profiles induced by $(\bar{s}_1^*, \bar{s}_2^{**})$ and $(\bar{s}_1^{**}, \bar{s}_2^*)$ are subgame-perfect equilibria of the subgame (V_1, V_2, K) and all its subgames starting at a non-terminal history.

When $V_1 = V_2$, there are multiple equilibria in the game. In fact, more equilibria can be obtained by changing agents' choices between indifferent alternatives at some histories.

We next note that the arguments in the proof of Proposition 1 in Compte and Jehiel [2003] show the following.

Fact 2. When $V_i < V_j$, the strategy profile $(\bar{s}_i^*, \bar{s}_j^{**})$ is a subgame-perfect equilibrium of the game with the restrictions $C_1 = C_2 = 0$. Accordingly, the strategy profile induced by $(\bar{s}_i^*, \bar{s}_j^{**})$ is a subgame-perfect equilibrium of the subgame (V_1, V_2, K) and all its subgames starting at a non-terminal history.

In this case $V_i < V_j$ also, there are multiple equilibria in the game; other equilibria can be obtained by changing agents' choices between indifferent alternatives off the equilibrium paths.

However, the equilibrium path is unique and common among all the subgame-perfect equilibria particularly when $V_i < V_j < K < \delta V_i + V_j$. Arguments on the equilibrium path are summarized in Facts 3 through 8; these facts are obtained in the proof of Proposition 1 in Compte and Jehiel [2003].

Fact 3. For each agent i, if $0 < x^{t-1} < (1-\delta)V_i$ for a subgame starting at a non-terminal history h^t with $i = m(t)$, then for any subgame-perfect equilibrium of the subgame, the equilibrium path is such that $c_i^t = x^{t-1}$, and the period of completion is t.

Fact 4. When $V_i < V_j$, if $(1-\delta)V_i < x^{t-1} < V_j$ for a subgame starting at a non-terminal history h^t with $j = m(t)$, then for any subgame-perfect equilibrium of the subgame, the equilibrium path is such that $c_j^t = x^{t-1}$, and the period of completion is t.

Fact 5. When $V_i < V_j$, if $(1-\delta)V_i < x^{t-1} < V_j$ for a subgame starting at a non-terminal history h^t with $i = m(t)$, then for any subgame-perfect equilibrium of the subgame, the equilibrium path is such that $c_i^t = 0$ and $c_j^{t+1} = x^{t-1}$, and the period of completion is $t+1$.

Fact 6. When $V_i < V_j$, if $V_j < x^{t-1} < \delta V_i + V_j$ for a subgame starting at a non-terminal history h^t with $i = m(t)$, then for any subgame-perfect equilibrium of the subgame, the equilibrium path is such that $c_i^t = x^{t-1} - V_j$ and $c_j^{t+1} = V_j$, and the period of completion is $t+1$.

Fact 7. When $V_i < V_j$, if $V_j < x^{t-1} < \delta V_i + V_j$ for a subgame starting at a non-terminal history h^t with $j = m(t)$, then for any subgame-perfect equilibrium of the subgame, the equilibrium path is such that $c_j^t = 0$, $c_i^{t+1} = x^{t-1} - V_j$, and $c_j^{t+2} = V_j$, and the period of completion is $t+2$.

Fact 8. When $V_i < V_j$, if $\delta V_i + V_j < x^{t-1}$ for a subgame starting at a non-terminal history h^t, then for any subgame-perfect equilibrium of the subgame, the equilibrium path is such that $c_i^\tau = c_j^\tau = 0$ for all $\tau \geq t$, i.e., no agent contributes a positive amount thereafter and the project is not completed forever.

We note that when the equilibrium path is unique, the equilibrium payoff profile is uniquely determined. Particularly, when Facts 6 and 7 apply, the equilibrium payoff profile is the following.

Fact 9. If $V_1 < V_2 < K < \delta V_1 + V_2$, the equilibrium payoff profile (U_1^*, U_2^*) is such that $(U_1^*, U_2^*) = (\delta V_1 + V_2 - K, 0)$, which is realized along the equilibrium path, $c_1^1 = K - V_2$ and $c_2^2 = V_2$, with the period of completion $T = 2$.

Fact 10. If $V_2 < V_1 < K < V_1 + \delta V_2$, the equilibrium payoff profile (U_1^*, U_2^*) is such that $(U_1^*, U_2^*) = (0, \delta V_1 + \delta^2 V_2 - \delta K)$, which is realized along the equilibrium path, $c_1^1 = 0$, $c_2^2 = K - V_1$, and $c_1^3 = V_1$, with the period of completion $T = 3$.

Before closing this section, we point out two properties of the equilibria of the game with the restrictions $C_1 = C_2 = 0$.

First, it is possible that the project is not completed even if completion of the project is socially desirable. That is, the project is not completed when $V_i < V_j < \delta V_i + V_j < K < V_1 + V_2$.

Second, the agent with the lower valuation gets all the surplus while the agent with the higher valuation gets a payoff of zero. That is, agent 2's payoff is zero when $V_1 < V_2$ by Fact 9 while agent 1's payoff is zero when $V_2 < V_1$ by Fact 10.

4. The Results for the One-Side Case

We prepare two lemmas before stating our propositions in this section. We note that these lemmas hold even when $C_1 \geq 0$ and $C_2 \geq 0$, i.e., both agents have options to make a proposal to commit to the final step. These lemmas will be used also in the next section.

Lemma 1. *For any strategy profile (s_1, s_2) which realizes a terminal history $h^{T+1} = \{(C_1, C_2), (c_1^0, c_2^0), \ldots, (c_1^T, c_2^T)\}$, the payoff profile $(u_1(s_1, s_2), u_2(s_1, s_2))$ satisfies the following:*

$$u_1(s_1, s_2) + u_2(s_1, s_2) \leq \delta^{T-1}(V_1 + V_2 - K).$$

Proof. Since completion of the project is socially desirable by the assumption of our model, we have $V_1 + V_2 - K > 0$. If $T = \infty$, i.e., if the project is not completed forever, then each agent i's payoff $u_i(s_1, s_2)$ cannot be positive and hence the inequality in the lemma clearly holds.

If $T < \infty$, then we have

$$\begin{aligned} u_1(s_1, s_2) + u_2(s_1, s_2) &= \delta^{T-1}(V_1 + V_2 - C_1 - C_2) - \sum_{t=1}^{T} \delta^{t-1}(c_1^t + c_2^t) \\ &\leq \delta^{T-1}(V_1 + V_2 - C_1 - C_2) - \sum_{t=1}^{T} \delta^{T-1}(c_1^t + c_2^t) \\ &= \delta^{T-1}(V_1 + V_2 - C_1 - C_2 - \sum_{t=0}^{T}(c_1^t + c_2^t)) \\ &\leq \delta^{T-1}(V_1 + V_2 - K) \end{aligned}$$

since $\delta < 1$, $c_1^t \geq 0$, $c_2^t \geq 0$, $c_1^0 = c_2^0 = 0$, and $x^T = K - C_1 - C_2 - \sum_{\tau=0}^{T}(c_1^\tau + c_2^\tau) \leq 0$. □

Lemma 2. *For any subgame-perfect equilibrium (s_1^*, s_2^*) of the game, the equilibrium payoffs $u_1(s_1^*, s_2^*)$ and $u_2(s_1^*, s_2^*)$ are non-negative.*

Proof. If agent i chooses a strategy s_i such that $s_i(h^t) = 0$ for any non-terminal history h^t, then agent i can secure a payoff of zero, regardless of his opponent's strategy. So, we have $u_i(s_i^*, s_j^*) \geq u_i(s_i, s_j^*) \geq 0$. □

We now investigate our model with the restriction in which only one agent has an option to make a proposal to commit to the final step. We first consider the game where only agent 1 has the option. That is, the admissible strategies for agent 1 are such that $C_1 \geq 0$ while the admissible strategies for agent 2 are restricted in such a way that $C_2 = 0$. Our proposition says that the equilibrium payoff profile and the equilibrium path are uniquely determined, regardless of whether $V_1 < V_2$ or not.

Proposition 1. *If there is any subgame-perfect equilibrium (s_1^*, s_2^*) in the game with the restrictions $C_1 \geq 0$ and $C_2 = 0$, the equilibrium payoff profile $(U_1^*, U_2^*) \equiv (u_1(s_1^*, s_2^*), u_2(s_1^*, s_2^*))$ is such that $(U_1^*, U_2^*) = (\delta(V_1 + V_2 - K), 0)$, which is realized along the equilibrium path, $C_1 = K - V_2$, $c_1^1 = 0$, and $c_2^2 = V_2$, with the period of completion $T = 2$.*

Proof. Since we have the restriction $C_2 = 0$, i.e., since agent 2 does not have an opportunity to contribute before period 2, if ever $T = 1$, then it must be the case that agent 1 bears all the cost of the project, which makes agent 1's payoff negative since $V_1 < K$. So, we have $T \geq 2$ in equilibrium by Lemma 2, and hence $U_1^* + U_2^* \leq \delta(V_1 + V_2 - K)$ by Lemma 1 and our assumption $\delta < 1$.

We now choose $\varepsilon > 0$ and let $C_1 = K - V_2 + \varepsilon$, $\bar{V}_1 = V_1 - C_1$, and $\bar{K} = K - C_1$. Since $V_1 < K < V_1 + V_2$ by our assumption, we can choose sufficiently small ε so that $0 < \bar{V}_1 < V_2$ and $(1 - \delta)\bar{V}_1 < \bar{K} < V_2$.

Consider a subgame-perfect equilibrium of the subgame $(\bar{V}_1, V_2, \bar{K})$ starting in period 1. We note that the previous results in Section 3 apply to the subgame; the remaining amount required for completion is $\bar{K} > 0$, and on completion of the project, agent 1 obtains a *net* benefit $\bar{V}_1 > 0$ while agent 2 obtains a benefit $V_2 > 0$. By Fact 5, the equilibrium path in the subgame is such that $c_1^1 = 0$ and $c_2^2 = \bar{K}$, and hence agent 1's equilibrium payoff U_1^{**} is such that $U_1^{**} = \delta\bar{V}_1 = \delta(V_1 + V_2 - K - \varepsilon)$.

Next, consider the subgame-perfect equilibrium (s_1^*, s_2^*) of the whole game. Since agent 1 has an option to choose $C_1 = K - V_2 + \varepsilon$ in period 0, his equilibrium payoff U_1^* must be such that $U_1^* \geq U_1^{**}$. Since we can have $\varepsilon > 0$ arbitrarily close to 0, it must be the case that $U_1^* \geq \delta(V_1 + V_2 - K)$.

Considering the above inequalities, $U_1^* + U_2^* \leq \delta(V_1 + V_2 - K)$, $U_1^* \geq \delta(V_1 + V_2 - K)$, and $U_2^* \geq 0$ by Lemma 2, we obtain the result $(U_1^*, U_2^*) = (\delta(V_1 + V_2 - K), 0)$. Furthermore, we obtain $T = 2$ by Lemma 1 and our assumption $\delta < 1$. The results $U_2^* = 0$ and $T = 2$ imply that $c_2^2 = V_2$ on the equilibrium path.

We are left to show $C_1 = K - V_2$ and $c_1^1 = 0$ on the equilibrium path. Since $T = 2$ and $c_2^2 = V_2$, we have $C_1 + c_1^1 \geq K - V_2$. We recall that $U_1^* = \delta(V_1 - C_1) - c_1^1$. By the optimality of agent 1's choices on the equilibrium path, we have $C_1 + c_1^1 = K - V_2$. So, $U_1^* = \delta(V_1 + V_2 - K) - (1-\delta)c_1^1$, which gets bigger as c_1^1 gets smaller, and hence the inequality $c_1^1 \geq 0$ must bind. Therefore, $C_1 = K - V_2$ and $c_1^1 = 0$ are the optimal choices for agent 1. □

We next consider the game where only agent 2 has an option to propose.

Proposition 2. *If there is any subgame-perfect equilibrium (s_1^*, s_2^*) in the game with the restrictions $C_1 = 0$ and $C_2 \geq 0$, the equilibrium payoff profile $(U_1^*, U_2^*) \equiv (u_1(s_1^*, s_2^*), u_2(s_1^*, s_2^*))$ is such that $(U_1^*, U_2^*) = (0, V_1 + V_2 - K)$, which is realized along the equilibrium path, $C_2 = K - V_1$ and $c_1^1 = V_1$, with the period of completion $T = 1$.*

Proof. By Lemma 1 and our assumption $\delta < 1$, we have $U_1^* + U_2^* \leq V_1 + V_2 - K$.

We now choose $\varepsilon > 0$ and let $C_2 = K - V_1 + \varepsilon$, $\bar{V}_2 = V_2 - C_2$, and $\bar{K} = K - C_2$. Since $V_2 < K < V_1 + V_2$ by our assumption, we can choose sufficiently small ε so that $0 < \bar{V}_2 < V_1$ and $(1-\delta)\bar{V}_2 < \bar{K} < V_1$.

Consider a subgame-perfect equilibrium of the subgame $(V_1, \bar{V}_2, \bar{K})$ starting in period 1. We note that the previous results in Section 3 apply to the subgame; the remaining amount required for completion is $\bar{K} > 0$, and on completion of the project, agent 1 obtains a benefit $V_1 > 0$ while agent 2 obtains a *net* benefit $\bar{V}_2 > 0$. By Fact 4, the equilibrium path in the subgame is such that $c_1^1 = \bar{K}$, and hence agent 2's equilibrium payoff U_2^{**} is such that $U_2^{**} = \bar{V}_2 = V_1 + V_2 - K - \varepsilon$.

Next, consider the subgame-perfect equilibrium (s_1^*, s_2^*) of the whole game. Since agent 2 has an option to choose $C_2 = K - V_1 + \varepsilon$ in period 0, his equilibrium payoff U_2^* must be such that $U_2^* \geq U_2^{**}$. Since we can have $\varepsilon > 0$ arbitrarily close to 0, it must be the case that $U_2^* \geq V_1 + V_2 - K$.

Considering the above inequalities, $U_1^* + U_2^* \leq V_1 + V_2 - K$, $U_2^* \geq V_1 + V_2 - K$, and $U_1^* \geq 0$ by Lemma 2, we obtain the result $(U_1^*, U_2^*) = (0, V_1 + V_2 - K)$. Furthermore, we obtain $T = 1$ by Lemma 1 and our assumption $\delta < 1$. The results $(U_1^*, U_2^*) = (0, V_1 + V_2 - K)$ and $T = 1$ imply that $c_1^1 = V_1$ and $C_2 = K - V_1$ on the equilibrium path. □

Propositions 1 and 2 assume the existence of a subgame-perfect equilibrium (s_1^*, s_2^*). We next show that the following strategy profile $(\hat{s}_i^*, \hat{s}_j^{**})$ is in fact a subgame-perfect equilibrium of the game where agent i can choose $C_i \geq 0$ in period 0 while agent $j \neq i$ cannot.

Suppose that, for $t \geq 1$, we have a non-terminal history $h^t = \{(C_1, C_2), (c_1^0, c_2^0), \ldots, (c_1^{t-1}, c_2^{t-1})\}$ with the restrictions $C_i \geq 0$ and $C_j = 0$. We let $\bar{V}_i = V_i - C_i$ for notational convenience.

Strategy $\hat{s}_i^*$ for agent i in the game with the restrictions $C_i \geq 0$, $C_j = 0$, and $i \neq j$.

Let $\hat{s}_i^*(h^0) = K - V_j$. When $i \neq m(t)$, we require that $\hat{s}_i^*(h^t) = 0$ by the feasibility for contributions. When $i = m(t)$, the following descriptions define $\hat{s}_i^*(h^t)$.

(i) If $x^{t-1} \leq 0$, then let $\hat{s}_i^*(h^t) = 0$.

(ii) If $\bar{V}_i \leq V_j$ and $0 < x^{t-1} \leq (1-\delta)\bar{V}_i$, then let $\hat{s}_i^*(h^t) = x^{t-1}$.

(iii) If $\bar{V}_i \leq V_j$ and $(1-\delta)\bar{V}_i < x^{t-1} \leq V_j$, then let $\hat{s}_i^*(h^t) = 0$.

(iv) If $\bar{V}_i \leq V_j$ and $V_j < x^{t-1} \leq \delta\bar{V}_i + V_j$, then let $\hat{s}_i^*(h^t) = x^{t-1} - V_j$.

(v) If $\bar{V}_i \leq V_j$ and $\delta\bar{V}_i + V_j < x^{t-1}$, then let $\hat{s}_i^*(h^t) = 0$.

(vi) If $V_j < \bar{V}_i$ and $0 < x^{t-1} \leq \bar{V}_i$, then let $\hat{s}_i^*(h^t) = x^{t-1}$.

(vii) If $V_j < \bar{V}_i$ and $\bar{V}_i < x^{t-1}$, then let $\hat{s}_i^*(h^t) = 0$.

Strategy $\hat{s}_j^{**}$ for agent j in the game with the restrictions $C_i \geq 0$, $C_j = 0$, and $i \neq j$.

We fix $\hat{s}_j^{**}(h^0) = 0$ by the restriction $C_j = 0$. When $j \neq m(t)$, we require that $\hat{s}_j^{**}(h^t) = 0$ by the feasibility for contributions. When $j = m(t)$, the following descriptions define $\hat{s}_j^{**}(h^t)$.

(i) If $x^{t-1} \leq 0$, then let $\hat{s}_j^{**}(h^t) = 0$.

(ii) If $V_j < \bar{V}_i$ and $0 < x^{t-1} \leq (1-\delta)V_j$, then let $\hat{s}_j^{**}(h^t) = x^{t-1}$.

(iii) If $V_j < \bar{V}_i$ and $(1-\delta)V_j < x^{t-1} \leq \bar{V}_i$, then let $\hat{s}_j^{**}(h^t) = 0$.

(iv) If $V_j < \bar{V}_i$ and $\bar{V}_i < x^{t-1} \leq \delta V_j + \bar{V}_i$, then let $\hat{s}_j^{**}(h^t) = x^{t-1} - \bar{V}_i$.

(v) If $V_j < \bar{V}_i$ and $\delta V_j + \bar{V}_i < x^{t-1}$, then let $\hat{s}_j^{**}(h^t) = 0$.

(vi) If $\bar{V}_i \leq V_j$ and $0 < x^{t-1} \leq V_j$, then let $\hat{s}_j^{**}(h^t) = x^{t-1}$.

(vii) If $\bar{V}_i \leq V_j$ and $V_j < x^{t-1}$, then let $\hat{s}_j^{**}(h^t) = 0$.

Proposition 3. *The strategy profile $(\hat{s}_i^*, \hat{s}_j^{**})$ is a subgame-perfect equilibrium of the game with the restrictions $C_i \geq 0$, $C_j = 0$, and $i \neq j$.*

Proof. Suppose that any $C_i \geq 0$ is given and $C_j = 0$ is fixed. Let $\bar{V}_i = V_i - C_i$ and $\bar{K} = K - C_i$. Take any non-terminal history h^t with $t \geq 1$. We will investigate the subgame starting at h^t in the following three cases.

First, consider the case where $x^{t-1} \leq 0$. If $t \geq 2$, then the project is completed in period $t-1$ and the game ends at the period, so no subgame starts at h^t. If $t = 1$, it is optimal for agent 1 to choose $c_1^1 = 0$ at h^1 because choosing $c_1^1 > 0$ simply lowers his payoff, considering that the project is completed and the game ends even if he contributes nothing in period 1. This optimal choice for agent 1 is described in Item (i) of the strategy $\hat{s}_1^*$ or $\hat{s}_1^{**}$. So, the strategy profile induced by $(\hat{s}_i^*, \hat{s}_j^{**})$ is a subgame-perfect equilibrium of the subgame starting at h^1, no matter whether $i = 1$ or $j = 1$.

Second, consider the case where $x^{t-1} > 0$ and $\bar{V}_i \leq 0$. Since agent i obtains a non-positive net benefit on completion of the project, it is optimal for agent i to contribute nothing at h^t and its continuation histories. This optimal choice for agent i is described in the strategy $\hat{s}_i^*$; Item (iii) or (v) in the descriptions of $\hat{s}_i^*$ applies here.[4] On the other hand, agent j obtains a positive benefit V_j on completion of the project. Since his opponent is expected to contribute nothing in the future, if agent j is on the move at h^t or its continuation history, it is optimal for him to contribute enough and complete the project immediately as long as the remaining amount does not exceed V_j. This optimal choice for agent j is described in the strategy $\hat{s}_j^{**}$; Item (vi) or (vii) in the descriptions of $\hat{s}_j^{**}$ applies. Therefore, the strategy profile induced by $(\hat{s}_i^*, \hat{s}_j^{**})$ is a subgame-perfect equilibrium of the subgame starting at h^t.

Third, consider the case where $x^{t-1} > 0$ and $\bar{V}_i > 0$. In this case, the facts discussed in the previous section apply to the subgame starting at h^t. Particularly, we focus on Facts 1 and 2 and the strategy profile $(\bar{s}_i^*, \bar{s}_j^{**})$ in the previous section. When $\bar{V}_i \leq V_j$, the strategy $\hat{s}_i^*$ is defined in the same way as $\bar{s}_i^*$ while the strategy $\hat{s}_j^{**}$ is defined in the same way as $\bar{s}_j^{**}$. When $V_j < \bar{V}_i$, the strategy $\hat{s}_i^*$ is defined in the same way as $\bar{s}_j^{**}$ while the strategy $\hat{s}_j^{**}$ is defined in the same way as $\bar{s}_i^*$. So, by Facts 1 and 2, the strategy profile induced by $(\hat{s}_i^*, \hat{s}_j^{**})$ is a subgame-perfect equilibrium of the subgame starting at h^t.

So far, we have shown that the strategy profile induced by $(\hat{s}_i^*, \hat{s}_j^{**})$ is a subgame-perfect equilibrium of the subgame starting at any non-terminal history h^t that continues after an arbitrary choice of $C_i \geq 0$. In other words, the strategy profile induced by $(\hat{s}_i^*, \hat{s}_j^{**})$ is a subgame-perfect equilibrium of any subgame starting in period 1. We note that, in the equilibrium of each subgame starting in period 1, agent j's payoff must be non-negative because agent j can secure a payoff of zero in the subgame by choosing to contribute nothing in the subgame.

We are left to show that $\hat{s}_i^*(h^0) = K - V_j$ is agent i's optimal choice in period 0. In doing so, we may assume that, in every subgame starting in period 1, agents i and j choose the strategy profile induced by $(\hat{s}_i^*, \hat{s}_j^{**})$.

Suppose that $i = 1$, i.e., it is agent 1 that has an option to make a proposal $C_1 \geq 0$. When $C_1 = K - V_2$, we have $\bar{K} = K - C_1 = V_2$, and $\bar{V}_1 = V_1 - C_1 < V_2$ by our assumption $V_1 < K$. In the

[4]When $\bar{V}_i \leq 0$, Items (ii), (iv), (vi), and (vii) in the descriptions of $\hat{s}_i^*$ never apply because the inequalities in the descriptions do not hold.

subgame $(\bar{V}_1, V_2, \bar{K})$ starting in period 1, the strategy profile induced by $(\hat{s}_1^*, \hat{s}_2^{**})$ realizes a path such that $c_1^1 = 0$ and $c_2^2 = \bar{K}$ with the period of completion $T = 2$, and agent 1's payoff along the path is $\delta \bar{V}_1 = \delta(V_1 + V_2 - K) > 0$. We investigate whether agent 1 can get a payoff higher than $\delta(V_1 + V_2 - K)$ by choosing $C_1 \neq K - V_2$. In the subgame starting in period 1 after the choice of C_1, agent 2's payoff is non-negative as we have mentioned above. By Lemma 1, agent 1's payoff realized in the subgame does not exceed $\delta^{T-1}(V_1 + V_2 - K)$. If $T \geq 2$ by the choice of C_1, agent 1's payoff does not exceed $\delta(V_1 + V_2 - K)$ since $\delta < 1$. Since agent 2 does not have an opportunity to contribute before period 2, if $T = 1$ by the choice of C_1, then it must be the case that agent 1 bears all the cost of the project, which makes agent 1's payoff negative since $V_1 < K$. Therefore, agent 1's payoff cannot get higher than $\delta(V_1 + V_2 - K)$ even if he chooses any $C_1 \neq K - V_2$. So, $\hat{s}_1^*(h^0) = K - V_2$ is agent 1's optimal choice.

Suppose that $i = 2$, i.e., it is agent 2 that has an option to make a proposal $C_2 \geq 0$. When $C_2 = K - V_1$, we have $\bar{K} = K - C_2 = V_1$, and $\bar{V}_2 = V_2 - C_2 < V_1$ by our assumption $V_2 < K$. In the subgame $(V_1, \bar{V}_2, \bar{K})$ starting in period 1, the strategy profile induced by $(\hat{s}_1^{**}, \hat{s}_2^*)$ realizes a path such that $c_1^1 = \bar{K}$ with the period of completion $T = 1$, and agent 2's payoff along the path is $\bar{V}_2 = V_1 + V_2 - K > 0$. We investigate whether agent 2 can get a payoff higher than $V_1 + V_2 - K$ by choosing $C_2 \neq K - V_1$. In the subgame starting in period 1 after the choice of C_2, agent 1's payoff is non-negative as we have mentioned above. By Lemma 1, agent 2's payoff realized in the subgame does not exceed $V_1 + V_2 - K$ since $\delta < 1$. Therefore, agent 2's payoff cannot get higher than $V_1 + V_2 - K$ even if he chooses any $C_2 \neq K - V_1$. So, $\hat{s}_2^*(h^0) = K - V_1$ is agent 2's optimal choice. □

In this section, we have considered the game where only one agent has an option to make a proposal to commit to the final step. We point out two properties of the equilibria of the game.

First, the project is completed in equilibrium as long as completion of the project is socially desirable: $K < V_1 + V_2$. This is in contrast with the fact that the project is not completed when $V_i < V_j < \delta V_i + V_j < K < V_1 + V_2$ in equilibrium of the game with the restrictions $C_1 = C_2 = 0$ in the previous section.

Second, the agent with the option gets all the surplus while the other agent without the option gets a payoff of zero. The option gives an advantage to the proposer because he comes to be able to manipulate his net valuation and make it lower than his opponent's valuation. The option also gives him a chance to postpone his actual payment, which is another advantage to him. Although a commitment to the final step seems a generous proposal at a glance, the commitment gives the proposer a great advantage in his equilibrium payoff.

5. The Results for the Two-Side Case

This section studies our model with the restriction in which both agents can make a proposal. The admissible strategies for agents 1 and 2 are such that $C_1 \geq 0$ and $C_2 \geq 0$. Define

$$\hat{V} = \frac{1}{1+\delta}(V_1 + V_2 - K)$$

for notational convenience. We have $0 < \hat{V} < V_i$ by our assumption $0 < V_i < K < V_1 + V_2$ for $i = 1, 2$.

Proposition 4. *If there is any subgame-perfect equilibrium (s_1^*, s_2^*) in the game with the restrictions $C_1 \geq 0$ and $C_2 \geq 0$, the equilibrium payoff profile $(U_1^*, U_2^*) \equiv (u_1(s_1^*, s_2^*), u_2(s_1^*, s_2^*))$ is such that $(U_1^*, U_2^*) = (\delta\hat{V}, \hat{V})$, which is realized along the equilibrium path, $C_1 = V_1 - \hat{V}$, $C_2 = V_2 - \hat{V}$, and $c_1^1 = K - C_1 - C_2 = (1-\delta)\hat{V}$, with the period of completion $T = 1$.*

Proof. The proposition is proved in four steps. Recall that we use the following notations: $\bar{V}_1 = V_1 - C_1$, $\bar{V}_2 = V_2 - C_2$, and $\bar{K} = K - C_1 - C_2$.

Step 1. $U_1^* \geq \delta\hat{V}$.

Proof of Step 1. We show that, for any value of $\bar{V}_2$, agent 1 can secure a payoff of $\delta\hat{V}$ by choosing $C_1 = V_1 - \hat{V}$, which means that when agent 1 optimally chooses C_1 in the equilibrium (s_1^*, s_2^*), his payoff U_1^* must be no less than $\delta\hat{V}$. We assume that the equilibrium payoff profile induced by (s_1^*, s_2^*) is realized in the subgame $(\bar{V}_1, \bar{V}_2, \bar{K})$ starting in period 1. Note that $\bar{V}_1 = \hat{V}$ for the choice of C_1.

If $\bar{V}_2 > \hat{V}$, we have $(1-\delta)\bar{V}_1 < \bar{K} < \bar{V}_2$. This is because $\bar{K} = K - (V_1 - \bar{V}_1) - (V_2 - \bar{V}_2) > K - (V_1 - \hat{V}) - (V_2 - \hat{V}) = (1-\delta)\hat{V} = (1-\delta)\bar{V}_1$ and $\bar{V}_2 - \bar{K} = V_1 + V_2 - K - \hat{V} = \delta\hat{V} > 0$. Since $0 < \bar{V}_1 < \bar{V}_2$ and $\bar{K} > 0$, the facts discussed in Section 3 apply to the subgame $(\bar{V}_1, \bar{V}_2, \bar{K})$ starting in period 1. By Fact 5, $c_1^1 = 0$ and $c_2^2 = \bar{K}$ with the period of completion $T = 2$ in equilibrium of the subgame, and hence agent 1's equilibrium payoff in the subgame is $\delta\bar{V}_1 = \delta\hat{V}$.

If $\bar{V}_2 \leq \hat{V}$, we have $\bar{K} \leq (1-\delta)\bar{V}_1$. If $\bar{K} > 0$, agent 1's equilibrium payoff in the subgame $(\bar{V}_1, \bar{V}_2, \bar{K})$ is no less than $\delta\hat{V}$ because agent 1 has an option to choose $c_1^1 = \bar{K}$, finish the game in $T = 1$, and obtain a payoff $\bar{V}_1 - \bar{K} \geq \bar{V}_1 - (1-\delta)\bar{V}_1 = \delta\bar{V}_1 = \delta\hat{V}$. If $\bar{K} \leq 0$, it is optimal for agent 1 to choose $c_1^1 = 0$, finish the game in $T = 1$, and obtain a payoff $\bar{V}_1 > \delta\hat{V}$ in the subgame.

Step 2. $U_2^* \geq \hat{V}$.

Proof of Step 2. Suppose that C_1 is agent 1's choice in the equilibrium (s_1^*, s_2^*). We assume that, in every subgame starting in period 1, the equilibrium payoff profile induced by (s_1^*, s_2^*) is realized.

When $\bar{V}_1 > \hat{V}$, consider agent 2's choice $C_2' = V_2 - \hat{V}$. Let $\bar{V}_2' = V_2 - C_2'$ and $\bar{K}' = K - C_1 - C_2'$. Note that $\bar{V}_2' = \hat{V}$ for the choice of C_2'. We have $(1-\delta)\bar{V}_2' < \bar{K}' < \bar{V}_1$ because $\bar{K}' = K - (V_1 - \bar{V}_1) - (V_2 - \hat{V}) > K - (V_1 - \hat{V}) - (V_2 - \hat{V}) = (1-\delta)\hat{V} = (1-\delta)\bar{V}_2'$ and $\bar{V}_1 - \bar{K}' = V_1 + V_2 - K - \hat{V} = \delta\hat{V} > 0$. Since $0 < \bar{V}_2' < \bar{V}_1$ and $\bar{K}' > 0$, the facts discussed in Section 3 apply to the subgame $(\bar{V}_1, \bar{V}_2', \bar{K}')$ starting in period 1. By Fact 4, $c_1^1 = \bar{K}'$ with the period of completion $T = 1$ in equilibrium of the subgame, and hence agent 2's equilibrium payoff in the subgame is $\bar{V}_2' = \hat{V}$. Since agent 2 can secure a payoff of $\hat{V}$ by the choice of C_2', when agent 2 optimally chooses C_2 in the equilibrium (s_1^*, s_2^*), we must have $U_2^* \geq \hat{V}$.

When $\bar{V}_1 \leq \hat{V}$, consider agent 2's choice $C_2' = V_2 - \hat{V} + \varepsilon$ where $0 < \varepsilon < \hat{V}$. Let $\bar{V}_2' = V_2 - C_2'$ and $\bar{K}' = K - C_1 - C_2'$. Note that $\bar{V}_2' = \hat{V} - \varepsilon > 0$. We have $\bar{K}' < (1-\delta)\bar{V}_1$ because $\bar{K}' = K - (V_1 - \bar{V}_1) - (V_2 - \hat{V} + \varepsilon) = \bar{V}_1 - \delta\hat{V} - \varepsilon < \bar{V}_1 - \delta\bar{V}_1 = (1-\delta)\bar{V}_1$. If $\bar{K}' > 0$, we have $\bar{V}_1 > 0$ and Fact 3 applies to the subgame $(\bar{V}_1, \bar{V}_2', \bar{K}')$ starting in period 1. So, we have $c_1^1 = \bar{K}'$ with the period of completion $T = 1$ in equilibrium of the subgame, and hence agent 2's equilibrium payoff in the subgame is $\bar{V}_2' = \hat{V} - \varepsilon$. If $\bar{K}' \leq 0$, then it is optimal for agent 1 to choose $c_1^1 = 0$ and finish the game in $T = 1$. So, agent 2's equilibrium payoff in the subgame is $\bar{V}_2' = \hat{V} - \varepsilon$. When we consider agent 2's equilibrium payoff U_2^* in the whole game, we must have $U_2^* \geq \hat{V} - \varepsilon$ since agent 2 has an option to choose $C_2' = V_2 - \hat{V} + \varepsilon$. Since we can have $\varepsilon > 0$ arbitrarily close to 0, it must be the case that $U_2^* \geq \hat{V}$.

Step 3. *In equilibrium,* $T = 1$, $U_1^* = \delta\hat{V}$, $U_2^* = \hat{V}$, *and* $C_2 = V_2 - \hat{V}$.

Proof of Step 3. Steps 1 and 2 imply that $U_1^* + U_2^* \geq (1+\delta)\hat{V} = V_1 + V_2 - K$ while Lemma 1 implies that $U_1^* + U_2^* \leq \delta^{T-1}(V_1 + V_2 - K)$. Since $\delta < 1$, we must have $T = 1$ in equilibrium and $U_1^* + U_2^* = V_1 + V_2 - K$. By Steps 1 and 2, we obtain $U_1^* = \delta\hat{V}$ and $U_2^* = \hat{V}$.

Since $T = 1$ in equilibrium, the result $U_2^* = \hat{V}$ implies that $C_2 = V_2 - \hat{V}$ in equilibrium.

Step 4. *In equilibrium,* $C_1 = V_1 - \hat{V}$ *and* $c_1^1 = (1-\delta)\hat{V}$.

Proof of Step 4. Suppose that C_1 is agent 1's choice in equilibrium. Let $\bar{V}_1 = V_1 - C_1$. By Step 3, we have $T = 1$ and $U_1^* = \delta\hat{V}$ in equilibrium. By the form of agent 1's payoff function, we have $U_1^* = \bar{V}_1 - c_1^1 = \delta\hat{V}$. Considering $c_1^1 \geq 0$, we have $\bar{V}_1 \geq \delta\hat{V}$. We will show $\bar{V}_1 = \hat{V}$ in equilibrium.

First, suppose, by way of contradiction, that $\bar{V}_1 < \hat{V}$ in equilibrium. Choose sufficiently small $\varepsilon > 0$ so that $\varepsilon < \delta(\hat{V} - \bar{V}_1)$ and $\varepsilon < V_2 - \hat{V}$. Let $C_2' = V_2 - \hat{V} - \varepsilon$, $\bar{V}_2' = V_2 - C_2'$, and $\bar{K}' = K - C_1 - C_2'$. Then, we have $\bar{V}_2' = \hat{V} + \varepsilon$ and $\bar{K}' = K - (V_1 - \bar{V}_1) - (V_2 - \hat{V} - \varepsilon) = \bar{V}_1 - \delta\hat{V} + \varepsilon \geq \varepsilon$. If agent 2 chooses C_2' in period 0, then he enters the subgame $(\bar{V}_1, \bar{V}_2', \bar{K}')$ starting in period 1, to which the facts discussed in Section 3 apply because $0 < \bar{V}_1 < \bar{V}_2'$ and $\bar{K}' > 0$. We note that $\bar{K}' < (1-\delta)\bar{V}_1$ since

$(1-\delta)\bar{V}_1 - \bar{K}' = \delta(\hat{V} - \bar{V}_1) - \varepsilon > 0$. By Fact 3, we have $c_1^1 = \bar{K}'$ with the period of completion $T = 1$ in equilibrium of the subgame $(\bar{V}_1, \bar{V}_2', \bar{K}')$, in which agent 2's equilibrium payoff is $\bar{V}_2' = \hat{V} + \varepsilon > \hat{V}$. This is in contradiction with the result $U_2^* = \hat{V}$ in Step 3.

Second, suppose, by way of contradiction, that $\bar{V}_1 > \hat{V}$ in equilibrium. Choose sufficiently small $\varepsilon > 0$ so that $\varepsilon < \delta\hat{V}$, $\varepsilon < \bar{V}_1 - \hat{V}$, and $\varepsilon < \bar{V}_2 - \hat{V}$. Let $C_2' = V_2 - \hat{V} - \varepsilon$, $\bar{V}_2' = V_2 - C_2'$, and $\bar{K}' = K - C_1 - C_2'$. Then, we have $\bar{V}_2' = \hat{V} + \varepsilon$ and $\bar{K}' = \bar{V}_1 - \delta\hat{V} + \varepsilon > (1-\delta)\hat{V} + \varepsilon$. Note that $\bar{V}_1 > \bar{V}_2'$ since $\bar{V}_1 - \bar{V}_2' = \bar{V}_1 - \hat{V} - \varepsilon > 0$. If agent 2 chooses C_2' in period 0, then he enters the subgame $(\bar{V}_1, \bar{V}_2', \bar{K}')$ starting in period 1, to which the facts discussed in Section 3 apply because $0 < \bar{V}_2' < \bar{V}_1$ and $\bar{K}' > 0$. We note that $(1-\delta)\bar{V}_2' < \bar{K}' < \bar{V}_1$ since $\bar{K}' - (1-\delta)\bar{V}_2' > (1-\delta)\hat{V} + \varepsilon - (1-\delta)(\hat{V} + \varepsilon) = \delta\varepsilon > 0$ and $\bar{V}_1 - \bar{K}' = \delta\hat{V} - \varepsilon > 0$. By Fact 4, we have $c_1^1 = \bar{K}'$ with the period of completion $T = 1$ in equilibrium of the subgame $(\bar{V}_1, \bar{V}_2', \bar{K}')$, in which agent 2's equilibrium payoff is $\bar{V}_2' = \hat{V} + \varepsilon > \hat{V}$. This is in contradiction with the result $U_2^* = \hat{V}$ in Step 3.

Therefore, we have $\bar{V}_1 = \hat{V}$ in equilibrium, which implies that $C_1 = V_1 - \hat{V}$. Furthermore, in equilibrium, we have $c_1^1 = (1-\delta)\hat{V}$ since $U_1^* = \bar{V}_1 - c_1^1 = \delta\hat{V}$. □

Proposition 4 assumes the existence of a subgame-perfect equilibrium (s_1^*, s_2^*). We next show that the following strategy profile $(\tilde{s}_i^*, \tilde{s}_j^{**})$ is in fact a subgame-perfect equilibrium of the game.

Suppose that, for $t \geq 1$, we have a non-terminal history $h^t = \{(C_1, C_2), (c_1^0, c_2^0), \ldots, (c_1^{t-1}, c_2^{t-1})\}$ with the restrictions $C_1 \geq 0$ and $C_2 \geq 0$. We use the following notations: $\bar{V}_1 = V_1 - C_1$, $\bar{V}_2 = V_2 - C_2$, and $\hat{V} = (V_1 + V_2 - K)/(1+\delta)$.

Strategy $\tilde{s}_i^*$ for agent i in the game with the restrictions $C_1 \geq 0$ and $C_2 \geq 0$.

Let $\tilde{s}_i^*(h^0) = V_i - \hat{V}$. When $i \neq m(t)$, we require that $\tilde{s}_i^*(h^t) = 0$ by the feasibility for contributions. When $i = m(t)$, the following descriptions define $\tilde{s}_i^*(h^t)$.

(i) If $x^{t-1} \leq 0$, then let $\tilde{s}_i^*(h^t) = 0$.

(ii) If $\bar{V}_i \leq \bar{V}_j$ and $0 < x^{t-1} \leq (1-\delta)\bar{V}_i$, then let $\tilde{s}_i^*(h^t) = x^{t-1}$.

(iii) If $\bar{V}_i \leq \bar{V}_j$ and $(1-\delta)\bar{V}_i < x^{t-1} \leq \bar{V}_j$, then let $\tilde{s}_i^*(h^t) = 0$.

(iv) If $\bar{V}_i \leq \bar{V}_j$ and $\bar{V}_j < x^{t-1} \leq \delta\bar{V}_i + \bar{V}_j$, then let $\tilde{s}_i^*(h^t) = x^{t-1} - \bar{V}_j$.

(v) If $\bar{V}_i \leq \bar{V}_j$ and $\delta\bar{V}_i + \bar{V}_j < x^{t-1}$, then let $\tilde{s}_i^*(h^t) = 0$.

(vi) If $\bar{V}_j < \bar{V}_i$ and $0 < x^{t-1} \leq \bar{V}_i$, then let $\tilde{s}_i^*(h^t) = x^{t-1}$.

(vii) If $\bar{V}_j < \bar{V}_i$ and $\bar{V}_i < x^{t-1}$, then let $\tilde{s}_i^*(h^t) = 0$.

Strategy $\tilde{s}_j^{**}$ for agent j in the game with the restrictions $C_1 \geq 0$ and $C_2 \geq 0$.

Let $\tilde{s}_j^{**}(h^0) = V_j - \hat{V}$. When $j \neq m(t)$, we require that $\tilde{s}_j^{**}(h^t) = 0$ by the feasibility for contributions. When $j = m(t)$, the following descriptions define $\tilde{s}_j^{**}(h^t)$.

(i) If $x^{t-1} \leq 0$, then let $\tilde{s}_j^{**}(h^t) = 0$.

(ii) If $\bar{V}_j < \bar{V}_i$ and $0 < x^{t-1} \leq (1-\delta)\bar{V}_j$, then let $\tilde{s}_j^{**}(h^t) = x^{t-1}$.

(iii) If $\bar{V}_j < \bar{V}_i$ and $(1-\delta)\bar{V}_j < x^{t-1} \leq \bar{V}_i$, then let $\tilde{s}_j^{**}(h^t) = 0$.

(iv) If $\bar{V}_j < \bar{V}_i$ and $\bar{V}_i < x^{t-1} \leq \delta\bar{V}_j + \bar{V}_i$, then let $\tilde{s}_j^{**}(h^t) = x^{t-1} - \bar{V}_i$.

(v) If $\bar{V}_j < \bar{V}_i$ and $\delta\bar{V}_j + \bar{V}_i < x^{t-1}$, then let $\tilde{s}_j^{**}(h^t) = 0$.

(vi) If $\bar{V}_i \leq \bar{V}_j$ and $0 < x^{t-1} \leq \bar{V}_j$, then let $\tilde{s}_j^{**}(h^t) = x^{t-1}$.

(vii) If $\bar{V}_i \leq \bar{V}_j$ and $\bar{V}_j < x^{t-1}$, then let $\tilde{s}_j^{**}(h^t) = 0$.

Proposition 5. *The strategy profile $(\tilde{s}_i^*, \tilde{s}_j^{**})$ is a subgame-perfect equilibrium of the game with the restrictions $C_1 \geq 0$ and $C_2 \geq 0$.*

Proof. Take any $C_1 \geq 0$ and $C_2 \geq 0$. We use the following notations: $\bar{V}_1 = V_1 - C_1$, $\bar{V}_2 = V_2 - C_2$, and $\bar{K} = K - C_1 - C_2$. We consider the subgame $(\bar{V}_1, \bar{V}_2, \bar{K})$ starting in period 1.

Let us compare the strategy profile $(\hat{s}_i^*, \hat{s}_j^{**})$ presented in Section 4 with the strategy profile $(\tilde{s}_i^*, \tilde{s}_j^{**})$ in this section. We note that the descriptions in Items (i) through (vii) are almost the same between $(\hat{s}_i^*, \hat{s}_j^{**})$ and $(\tilde{s}_i^*, \tilde{s}_j^{**})$ except for the point that V_j appears in $(\hat{s}_i^*, \hat{s}_j^{**})$ while $\bar{V}_j$ appears in $(\tilde{s}_i^*, \tilde{s}_j^{**})$. Although V_j is a positive number by the assumption of our model, $\bar{V}_j$ can be a non-positive number. We here recall that Proposition 3 in Section 4 implies that the strategy profile induced by $(\hat{s}_i^*, \hat{s}_j^{**})$ is a subgame-perfect equilibrium of any subgame $(\bar{V}_i, V_j, \bar{K})$ investigated in the previous section. So, if either $\bar{V}_1 > 0$ or $\bar{V}_2 > 0$ or both, then we can use the arguments in the proof of Proposition 3 and show that the strategy profile induced by $(\tilde{s}_i^*, \tilde{s}_j^{**})$ is a subgame-perfect equilibrium of the subgame $(\bar{V}_1, \bar{V}_2, \bar{K})$ investigated in this section. We do not repeat the arguments for the case: either $\bar{V}_1 > 0$ or $\bar{V}_2 > 0$ or both.

However, in this section, it is possible that $\bar{V}_1 \leq 0$ and $\bar{V}_2 \leq 0$. In this case, we must have $\bar{K} < 0$ since $\bar{K} = K - C_1 - C_2 = K - (V_1 - \bar{V}_1) - (V_2 - \bar{V}_2) \leq K - V_1 - V_2 < 0$, where the last inequality holds by our assumption that completion of the project is socially desirable. When $\bar{K} < 0$, the project is completed in period 1 and the game ends. It is optimal for agent 1 to choose $c_1^1 = 0$ because choosing $c_1^1 > 0$ simply lowers his payoff, considering that the project is completed and the game ends even if he contributes nothing in period 1. This optimal choice for agent 1 is described in Item (i) of the strategy $\tilde{s}_1^*$ or $\tilde{s}_1^{**}$. So, the strategy profile induced by $(\tilde{s}_i^*, \tilde{s}_j^{**})$ is a subgame-perfect equilibrium of the subgame $(\bar{V}_1, \bar{V}_2, \bar{K})$ also in this case: $\bar{V}_1 \leq 0$ and $\bar{V}_2 \leq 0$.

So far, we have shown that the strategy profile induced by $(\tilde{s}_i^*, \tilde{s}_j^{**})$ is a subgame-perfect equilibrium of the subgame $(\bar{V}_1, \bar{V}_2, \bar{K})$ that continues after arbitrary choices of $C_1 \geq 0$ and $C_2 \geq 0$. We are left to show that $\tilde{s}_i^*(h^0) = V_i - \hat{V}$ and $\tilde{s}_j^{**}(h^0) = V_j - \hat{V}$ are the optimal choices for agents i and j in period 0. When $C_1 = V_1 - \hat{V}$ and $C_2 = V_2 - \hat{V}$, we have $\bar{V}_1 = \bar{V}_2 = \hat{V} > \bar{K} = (1-\delta)\hat{V}$. In the subgame $(\bar{V}_1, \bar{V}_2, \bar{K})$ starting in period 1, the strategy profile induced by $(\tilde{s}_1^*, \tilde{s}_2^{**})$ as well as $(\tilde{s}_1^{**}, \tilde{s}_2^*)$ realizes a path such that $c_1^1 = \bar{K}$ with the period of completion $T = 1$, and agent 1's payoff along the path is $\bar{V}_1 - \bar{K} = \delta\hat{V}$ while agent 2's payoff along the path is $\bar{V}_2 = \hat{V}$. We investigate whether agent 1 or 2 can get a higher payoff if he unilaterally changes the choice of C_1 or C_2. In doing so, we may assume that, in every subgame starting in period 1, agents i and j choose the strategy profile induced by $(\tilde{s}_i^*, \tilde{s}_j^{**})$.

First, we investigate agent 1's deviation. Suppose that $C_2 = V_2 - \hat{V}$ is given. If agent 1 chooses C_1 such that $0 \leq C_1 < V_1 - \hat{V}$, then Item (vi) of the strategy $\tilde{s}_1^*$ or $\tilde{s}_1^{**}$ applies to agent 1 at the history h^1, with $c_1^1 = \bar{K}$ chosen, and the game ends in period 1; his payoff is $\bar{V}_1 - \bar{K} = (V_1 - C_1) - (K - C_1 - C_2) = V_1 - K + V_2 - \hat{V} = (1+\delta)\hat{V} - \hat{V} = \delta\hat{V}$. If agent 1 chooses C_1 such that $V_1 - \hat{V} < C_1 < V_1 - \delta\hat{V}$, then Item (ii) of the strategy $\tilde{s}_1^*$ or $\tilde{s}_1^{**}$ applies to agent 1 at the history h^1, with $c_1^1 = \bar{K}$ chosen, and the game ends in period 1; his payoff is $\bar{V}_1 - \bar{K} = \delta\hat{V}$. If agent 1 chooses C_1 such that $V_1 - \delta\hat{V} \leq C_1$, then Item (i) of the strategy $\tilde{s}_1^*$ or $\tilde{s}_1^{**}$ applies to agent 1 at the history h^1, with $c_1^1 = 0$ chosen, and the game ends in period 1; his payoff $\bar{V}_1 = V_1 - C_1$ does not exceed $\delta\hat{V}$. So, agent 1's payoff cannot be higher than $\delta\hat{V}$ even if he chooses $C_1 \geq 0$ such that $C_1 \neq V_1 - \hat{V}$.

Second, we investigate agent 2's deviation. Suppose that $C_1 = V_1 - \hat{V}$ is given. If agent 2 chooses C_2 such that $0 \leq C_2 < V_2 - \hat{V}$, then Item (iii) of the strategy $\tilde{s}_1^*$ or $\tilde{s}_1^{**}$ applies to agent 1 at the history h^1, with $c_1^1 = 0$ chosen, and Item (vi) of the strategy $\tilde{s}_2^{**}$ or $\tilde{s}_2^*$ applies to agent 2 at the history h^2, with $c_2^2 = \bar{K}$ chosen, and the game ends in period 2; agent 2's payoff is $\delta(\bar{V}_2 - \bar{K}) = \delta((V_2 - C_2) - (K - C_1 - C_2)) = \delta(V_2 - K + V_1 - \hat{V}) = \delta((1+\delta)\hat{V} - \hat{V}) = \delta^2\hat{V}$. If agent 2 chooses C_2 such that $V_2 - \hat{V} < C_2 < V_2 - \delta\hat{V}$, then Item (vi) of the strategy $\tilde{s}_1^*$ or $\tilde{s}_1^{**}$ applies to agent 1 at the history h^1, with $c_1^1 = \bar{K}$ chosen, and the game ends in period 1; agent 2's payoff $\bar{V}_2 = V_2 - C_2$ is less than $\hat{V}$. If agent 2 chooses C_2 such that $V_2 - \delta\hat{V} \leq C_2$, then Item (i) of the strategy $\tilde{s}_1^*$ or $\tilde{s}_1^{**}$ applies to agent 1 at the history h^1, with $c_1^1 = 0$ chosen, and the game ends in period 1; agent 2's payoff $\bar{V}_2 = V_2 - C_2$ does not exceed $\delta\hat{V}$. So, agent 2's payoff cannot be higher than $\hat{V}$ even if he chooses $C_2 \geq 0$ such that $C_2 \neq V_2 - \hat{V}$. □

In this section, we have considered the game where both agents have options to make a proposal to commit to the final step. We point out three properties of the equilibria of this game.

First, the project is completed in equilibrium as long as completion of the project is socially desirable: $K < V_1 + V_2$. This property is also held by the equilibria of the game where only one agent can make a proposal.

Second, both agents get positive payoffs. We can say that the surplus is split relatively fairly between the two agents in equilibrium of this game, compared to the game where only one agent or no agent can propose.

Third, the second mover has an advantage. That is, agent 1 obtains a payoff of $\delta\hat{V}$ while agent 2 obtains a payoff of $\hat{V}$ in equilibrium. As the agents become more patient, i.e., as δ gets close to 1, the second mover's advantage becomes less.

6. Conclusion

We have investigated a two-player contribution game similar to the one studied by Compte and Jehiel [2003] but different in that our game assumes that agents can commit to make a certain amount of payment at the final step of completing a joint project. We have analyzed how such commitments to the final step can affect the equilibrium payoff profile in the game. We have shown the following.

In equilibrium of the game studied by Compte and Jehiel [2003], the agent with the lower valuation of the project gets all the surplus generated by completion of the project while the agent with the higher valuation gets a payoff of zero.

However, in equilibrium of our game where only one agent has an option to propose, the proposer gets all the surplus while the other agent without the option gets a payoff of zero. The option gives an advantage to the proposer because he comes to be able to manipulate his net valuation and make it lower than his opponent's valuation. The option also gives him such a chance to postpone his actual payment, as is another advantage to him.

In equilibrium of our game where both agents can propose, the surplus is split relatively fairly between the two in the sense that the both agents get positive payoffs, although the second mover's payoff exceeds that of the first mover.

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