Hedge Fund Contagion during the Financial Crisis

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Abstract

In this study, we investigate the volatility behavior of daily returns of hedge fund strategy indices, especially focusing on the inter-strategy contagion in the left-hand tail events by using a Markov regime-switching model. We find strong evidence of switching behavior in hedge fund index returns and that the short-lived hedge fund contagion occurred three times during the financial crisis of 2007-2009. These contagions were linked to specific crisis episodes: Quants Meltdown in August 2007, Bear Sterns collapse in March 2008, and collapse of Lehman Brothers in September 2008. On these occasions, Macro/CTA plays a significant role in the emerging hedge fund contagion and is short-lived. As hedge fund contagion is captured by the coincidence of being in the high-volatility state among hedge fund indices, tail dependence (i.e., the correlation structures of the probability of being in a high-volatility state) should be examined. The correlations among Equity Hedge, Event Driven, and Relative Value Arbitrage tend to increase during crisis periods. Conversely, the correlations of Macro/CTA with the other three strategies decreased during crisis periods. Thus, Macro/CTA may offer effective protection against systemic risk by shortening the duration of the inter-strategy hedge fund contagion.

Key words: *hedge fund indices, Markov regime switching model, left-hand tail events, hedge fund contagion, financial crisis*

Contents

1. Introduction

Owing to the innovative and competitive nature of investment strategies, hedge funds have grown rapidly and spread into financial markets over the last three decades. The unique value of hedge funds is their ability to create an attractive risk-return profile, independent of financial market trends. Many hedge fund strategies are designed to produce a limited correlation with the market. Consequently, diversifying against poor performance in equity bear markets is the primary motivation for hedge fund investment.

Studies on hedge funds have started with the basis of the classical linear factor model in the context of modern portfolio theory. Fung and Hsieh [1997] analyze hedge fund performance based on Sharpe's [1992] asset class factor model, and they first report that hedge fund returns typically have a low correlation with standard asset indices. Meanwhile, a considerable number of studies (e.g., Fung and Hsieh [1997], [2002], [2004]; Mitchel and Pulvino [2001]; Agarwal and Naik, [2004]) document nonlinear risk return characteristics of hedge fund strategies in the market. This ability arises from their dynamic strategies, such as leverage, short selling, derivatives, and switching investment tactics with changing market conditions. Consequently, hedge fund returns often exhibit a discontinuous shift in average returns and volatility in the data generating process. Chan et al. [2005] call this property "phase locking" behavior, a situation wherein otherwise uncorrelated actions suddenly become synchronized. Examples of such phenomena include the Mexican peso crisis of 1994-1995, the Asian currency crisis of 1997, the failure of Long-Term Capital Management of 1998, and the global financial crisis of 2007-2009.

A surge of interest in contagion in the literature (e.g., Eichengreen et al., [1996]; Dornbush et al., [2000]; Forbes and Rigobon, [2001], [2002]; Bae et al., [2003]; Dungey et al., [2004]) arises from a series of financial crises in the 1990s. These crises are characterized by the spread of market disturbances from an initial country-specific shock to markets worldwide. Bae et al. [2003] define financial contagion as the joint occurrence of extreme events across countries, which also captures the coincidence of extreme return shocks. Consequently, the role and impact of hedge funds on systemic risk is frequently raised during and after every financial crisis.

Numerous empirical studies have shed light on hedge fund contagion (Chan et al., [2005]; Boyson et al., [2006], [2010]; Li and Kazemi, [2007]; Billio et al., [2008], [2010]; Dudley and Nimalendran, [2011]: Akay et al., [2013]; Sawsen and Skander, [2016]; Kim and Lee, [2018]; Lee and Kim, [2018]; Sias et al., [2018]). Although these studies examine hedge fund contagion from various context and use different methodologies, their main focus is broadly divided into two streams of contagious effects: (1) whether extreme movements, such as equity market crisis, are transmitted to hedge funds and (2) whether extreme adverse returns in one hedge fund style are contagious to other hedge fund styles (i.e., inter-strategy contagion). Inter-strategy contagion implies that poor performance in one hedge fund strategy quickly transmits to the entire hedge fund industry. Boyson et al. [2006] are the first to investigate hedge fund contagion using daily data on hedge fund returns. They consider these aspects of hedge fund contagion by employing binomial and multinomial logit models. In particular, they focus on contagious effects of extreme events. However, they empirically find little evidence of contagion from financial markets to hedge funds, suggesting a diversification effect from hedge fund investments. By contrast, they find strong evidence of contagion among extremely poor returns of hedge fund styles. This indicates that downside risk is not reduced despite that hedge fund styles within a portfolio are diversified.[1)] When we consider the potential impact of hedge funds on systemic risk in financial markets, inter-strategy contagion adds a negative disruptive force to systemic risk, especially revealed during worst timing (e.g., during crisis periods). Therefore, this study focuses on inter-strategy contagion.

Our study is closely related to those of Chan et al. [2005] and Billio et al. [2008, 2010], which investigate the presence of the phase-locking behavior of hedge fund returns and contagion among hedge funds based on the regime-switching approach. Boyson et al. [2006] define contagion as a phenomenon that, during recession, assets tend to move together more closely than would be predicted through correlations. Therefore, hedge fund contagion focuses on left-tail events, which measure the nonlinear effect of tail dependencies between hedge funds. Billio et al. [2008] define hedge fund contagion as a significant increase in the joint probability of being in a high-volatility state among each strategy in the context of the regime-switching approach. This approach allows us to identify whether switching to the high-volatility regime coincides with a specific financial crisis.

Since the seminal work of Hamilton [1989], regime-switching models have become popular in econometric modeling. Their attractive features for application in financial modeling are aptly summarized by Ang and Timmermann [2011]. First, the notion of regime changes is natural and intuitive, and is closely linked to different periods in regulation, policy, and good or bad market conditions. Second, regime-switching models can parsimoniously capture stylized facts of many financial return series, including fat tails, heteroskedasticity, skewness, and time-varying correlations. The statistical properties of hedge fund index returns in this study retain these stylized facts (Munechika, [2021]). Third, regime-switching models construct nonlinear stylized dynamics of asset returns in a framework based on linear specifications within a regime.

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1) Li and Kazemi [2007] use daily hedge fund returns and conclude the absence of evidence supporting contagion between hedge funds and other investment in extreme down versus extreme up markets and between various hedge fund strategies themselves. Li and Kazemi [2007] also mention that their results are not directly comparable to those of Boyson et al. [2006] due to the methodology differences on how hedge fund contagion is examined. They focus on asymmetry in conditional correlations and claim that some of their findings are consistent with Boyson et al. [2006].

Several methodologies associated with structural changes exist, such least squares with breakpoint and threshold regression. The most significant factor is that they require a priori dating of crisis episodes. Hence, the Markov regime-switching model is a more flexible approach that does not require a priori dating of crisis and regards the structural change as a random event. Hence, the advantage of this methodology is that it allows for an endogenous definition of structural breaks. In addition, Billio et al. [2008] noted that this approach enables us to consider the cluster effect of the presence of persistent regimes and provide an accurate representation of the left-hand tail of the return distribution and, consequently, capture the phase-locking property of hedge fund returns. Therefore, applying the regime-switching model to hedge fund returns is motivated by the presence of these properties of the return-generating process and is based on the notion that the return distribution depends on the state. The change in underlying state probabilities over time leads to time-varying expected returns, volatility persistence, and changing correlations. Thus, the model can capture and identify regimes with quite different means and volatilities.

Most previous studies that have applied a regime-switching approach to hedge fund analysis (e.g., Alexander and Dimitriu, [2005]; Chan, et al., [2005]; Billio, et al., [2008], [2010]; Akay, et al., [2013]; Blazsek and Dowarowicz, [2011]; Sawsen and Skander, [2016]) are based on data reported at a monthly frequency. Relatively, few studies are based on daily data sets (Bock and Mestel, [2008]; Saunders et al., [2013]; Luo et al., [2015]). Specifying the number of regimes is a crucial issue that strongly affects the findings of research. In addition to selecting the number of regimes, the difference in data frequency influences the empirical results. Different sampling frequencies of time series data may exhibit varying properties despite that the data are obtained from the same data-generating process. Clark [1973] documents that the return has leptokurtic as the interval of the return becomes short and the number of observations increases. Munechika [2021] confirms that all distributions of hedge fund index returns are heavier tailed than those of monthly returns because kurtosis increases from monthly returns to daily returns. Therefore, hedge fund contagion linked to specific episodes during a crisis period can be detected.

Considering the above discussion, we believe that the best methodology should allow the model to detect short-lived and infrequent events based on the return-generating process. That is, the conditional probabilities of a high-volatility state derived from the Markov regime switching model can be used to detect hedge fund contagion. In our framework, we define contagion among hedge fund strategies when we observe that all hedge fund strategies are simultaneously in a high-volatility state, statistically measured by the joint probabilities of a high-volatility state. Specifically, this endogenization of the identification of hedge fund contagion within our modeling procedures is a major feature of this study. This study aims to identify hedge fund tail risk and to detect short-lived contagion endogenously by employing the Markov regime-switching model.

The following are the two main contributions of this study to the literature. First, in the context of the regime-switching approach, using daily data represents an important innovation because earlier research on hedge fund contagion mainly uses monthly data only. The main benefits of using daily data are that short-lived hedge fund contagions during the financial crisis of 2007-2009 can be detected and insights into each timing and duration can be provided. This study identifies hedge fund contagions that are clearly linked with crisis episodes during the financial crisis of 2007-2009. The first contagion coincides with the Quants Meltdown of August 2007, which lasted for two weeks. The second one is only a one-day, short-lived event during the Bear Sterns collapse in March 2008. The third one coincides with the collapse of Lehman Brothers in September 2008, which lasted, for nine days.

Second, the issue of why hedge fund contagions are short-lived events is closely related to tail dependence structure among the daily returns of each strategy. It is well known that the correlations of the return distributions tend to increase in their left-tails. However, the correlations of the probability of being in a high-volatility state of Macro/CTA and the other three strategies do not exhibit tail dependence. Thus, Macro/CTA may offer effective protection against systemic risk by shortening the duration of the inter-strategy hedge fund contagion.

The remainder of this paper is organized as follows. Section 2 introduces the model framework, Markov regime-switching approach. Section 3 describes the data and provides statistical properties. Then, the model is implemented and the estimated results are presented in Section 3. Section 4 examines the impact of hedge fund contagion. Finally, Section 5 concludes the paper.

2. Methodology

Dynamic time series models that involve unobserved variables are referred to as state space models, which were initially developed by control system engineers to measure a signal contained by noise (Harvey, A. C. [1981]). State-space analysis mainly aims to infer relevant properties of unobserved state variables from the knowledge of the observations of the time-series data. The key characteristics of the state variable are that it contains information from past and present data. However, the future behavior of the system is independent of the past values and depends only on the present values. This simplification expresses that knowledge of the state today is sufficient to anticipate the state tomorrow. Scientifically, the hidden state variable that characterizes the state dynamics follows a first-order Markov process. Kim and Nelson [1999] state that the state-space model with Markov switching may be considered a general approach to address endogenous structural breaks. That is, the Markov regime-switching model is a state-space model in which switching between regimes in its estimation occurs stochastically according to a Markov process. The model incorporates both (unobservable) state variables and regime switching.

2-1. Regime-switching model

The regime-switching approach presumes the existence of different states in financial modeling. For example, financial markets often switch from a low-volatility state to a high-volatility state and then back again. This demonstrates the importance of the concept of state in the time-series analysis. If the underlying data-generating process has a state structure, these states exhibit occasional jumps caused by structural breaks. When such a jump occurs, the distribution of the data-generating process changes. Negative skewed and fat-tailed distributions of hedge fund index returns can be represented using a mixture of simple underlying Gaussian distributions. Each of these Gaussian distributions corresponded to one regime.

Starting with the simplest specification, the returns of a hedge fund index r_t can be divided into two parts: the expected part of the return $E(r_t)$ and the unexpected part of the return ε_t:

$$r_t = E(r_t) + \varepsilon_t, \tag{1}$$

$$r_t = \mu + \varepsilon_t = \mu + \sigma z_t, \tag{2}$$

where μ denotes a mean and ε_t is a random innovation, $i.i.d.\sim N(0,\sigma^2)$, and z_t in the second equation is $i.i.d.\sim N(0,1)$.

With a regime-switching approach, equation (2) can be written as

$$r_t = \mu(s_t) + \varepsilon_t(s_t) = \mu(s_t) + \sigma(s_t)\, z_t, \tag{3}$$

where a state variable, $s_t \in \{j = 1,2,\cdots N\}$ indicates the regime at time t, and each state has its own mean and variance. The switching mean $\mu(s_t)$ and switching variance $\sigma(s_t)$ are the regime-dependent conditional mean and conditional variance. The observable variable r_t at time t is the actual data-generating process, which is determined by realizing the state variable s_t. However, they are not directly observable with a recognizable variable and are characterized by a hidden (or latent) variable. The change in regime is itself a random variable, and a first-order Markov process is used in characterizing the state dynamics. Therefore, the model includes a description of the probability law governing the change from state to state. The issues that we have to identify are identifying the state and understanding the dynamics of the timing of switches between states. Regarding the latter point, it is attractive to model such transitions as a Markov process because the timing of the switches between states is unknown. The regime itself is described as the outcome of an unobserved Markov chain.

A Markov chain can be used to govern the switches between regimes, where the hidden variable s_t is characterized by the state dynamics by using a first-order Markov process. It supposes the probability that s_t

equals a particular value j depending on the past only through the most recent value s_{t-1}.

$$P_r\{s_t = j|s_{t-1} = i,\ \ s_{t-2} = k, \ldots\} = P_r\{s_t = j|s_{t-1} = i\} = p_{ij} \tag{4}$$

The dynamics behind the switching process are driven by a transition matrix that collects the transition probabilities in an $(N \times N)$ matrix $\boldsymbol{P}$.

$$\boldsymbol{P} = \begin{bmatrix} p_{11} & \cdots & p_{1N} \\ \vdots & \ddots & \vdots \\ p_{N1} & \cdots & p_{NN} \end{bmatrix}, \tag{5}$$

where $p_{ij} = P_r\{s_t = j\,|s_{t-1} = i\}$ with $\sum_{j=1}^{N} p_{ij} = 1$ for all i. The transition probabilities of the Markov chain link each state in the chain to the next. The transition matrix $\boldsymbol{P}$ denotes the probability of the moving regimes. For example, the row 1 and column 2, p_{12}, provide the probability of state transition from state 1 to state 2 in the next time step. Parameters p_{11}, p_{22}, $p_{3,3}$, determine the probability of remaining in the same regime. This is one of the central points of the structure of a Markov regime-switching model.

The expected duration of staying in a certain regime j can be calculated using transition probabilities:

$$E(D_j) = \frac{1}{1-p_{jj}}, \quad j = 1,2,\cdots N, \tag{6}$$

where D_j denotes the duration of regime j.[2)] The higher the values of p_{jj}, the longer the expected duration of staying in a certain regime.

2-2. Three-state model

Specifying the number of regimes is crucial for the estimation of regime-switching models. However, the regime-switching process is difficult to select from the data based on simply performing statistical tests, such as likelihood ratio, Lagrange multiplier, or Wald tests. Moreover, in practical applications, distinguishing persistent level shifts from a single outlier is difficult, especially for a daily dataset. Ang and Timmermann [2011] state that the choice should be based on economic arguments as far as possible. Munechika [2021] provides a detailed specification analysis to determine the number of regimes for the same dataset used in this study. Considering a range of values for the number of states from two to four, the selected model with the most parsimonious number of parameters is a three-state model based on the information criteria and estimation results. Regimes are identified based on their volatility. In this study, we adopt the three-state Markov regime-switching model, in which three discrete states (i.e., s_1, s_2, s_3) represent (a) a high-volatility state, (b) a middle-volatility state, and (c) a low-volatility state, respectively. The higher value of p_{11}, p_{22}, and p_{33} are more likely to reject the null hypothesis of the no regime shift.

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2) Calculation of the expected duration is explained by Guo et al. [2011], p.98.

3. Model Implementation

3-1. Data description and statistical properties

The model introduced in section 2 is now implemented on hedge fund index returns. In the empirical investigation, the HFRX single strategy indices, on a daily basis, obtained from the Hedge Fund Research (HFR), Inc. database, were examined over the period from March 31, 2003, to March 16, 2017. The aggregation of the HFRX single strategy indices constitutes the HFRX Global Hedge Fund Index, which is designed to be representative of the overall composition of the hedge fund universe and to be investable. The underlying constituents and indices are assets weighted based on the distribution of assets in the hedge fund industry. Further, the HFRX single strategy indices correspond to four primary strategies: Equity Hedge, Event Driven, Macro/CTA, and Relative Value Arbitrage.[3)]

Table 1 reports the summary statistics and time series properties of the four hedge fund index returns. The returns are computed as log return, also called continuously compounded returns, denoted by r_t, and defined as

$$r_t = \log(P_t/P_{t-1}) * 100 = (p_t - p_{t-1}) * 100, \tag{7}$$

where P_t be the hedge fund index value at time t, and $p_t = \log(P_t)$ is called the log price.

First, a considerable heterogeneity exists in historical risk and return profiles. The best performing strategy in terms of returns is Event Driven. In terms of standard deviation, the lowest risk strategy is Relative Value Arbitrage, and the highest one is Equity Hedge.

Second, despite their heterogeneity, all indices share common characteristics: negative skewness,

Table 1：Summary statistics and time series properties of hedge fund index returns

	Equity Hedge	Event Driven	Macro/CTA	Relative Value Arbitrage
Mean	0.005	0.013	0.004	0.004
Std. Dev.	0.402	0.304	0.394	0.258
Skewness	-0.831	-1.043	-0.979	-1.619
Kurtosis	8.392	12.381	10.364	42.196
Jarque-Bera Test	4667.0***	13538.4***	8511.1***	226741.7***
LB(6)	91.307***	94.282***	40.604***	312.150***
$LB(6)^2$	1413.2***	1384.1***	949.0***	1463.6***
No. of observations	3518	3518	3518	3518

Note: The sample periods of returns are from April 1, 2003, to March 16, 2017.
The asterisks *, **, and *** denote significance at the 10%, 5%, and 1%levels, respectively.

3) To complete a fund profile for inclusion in the HFR subscriber database, an investment manager qualitatively chooses one of four strategies. The description of the four strategies is presented in Appendix.

leptokurtosis, and positive serial correlation. A negatively skewed distribution indicates that large negative returns are more common than large positive ones. When leptokurtosis is severe due to the high occurrence of outliers, these outliers have a powerful effect on the variance, and this small fraction inflate the variance. Specifically, "outlier-prone" probability distributions of hedge fund index returns indicate heavy tailed distributions. Heavy tails lead to extremely large returns (both negative and positive) occurring with a far greater frequency than normality would suggest. Therefore, negative skewed and heavy-tailed distributions indicate that hedge fund index returns frequently exhibit left-tail events. The non-normality of the return distributions is confirmed by normality tests based on Jarque-Bera statistics.

Table 1 also reports the Ljung-Box Q-statistic up to the sixth order in levels (LB(6)) and in squares of returns ($LB(6)^2$). When handling daily returns, the mean value is close to zero. Thus, the formula for variance can be approximated as squared returns. This clearly indicates that all indices exhibit serially correlated and volatility-clustered returns.

3-2. Estimated results

The estimated results of the three-state regime-switching model are listed in Table 2. We summarize some of our main empirical results. First, we find strong evidence of the switching behavior in the hedge fund index returns. All hedge fund indices clearly exhibit the relationship of the risk-return trade-off across the three regimes, which implies that the higher volatility state has lower expected returns. The first regime is identified as a high-volatility state with a large negative mean return. The second regime is characterized by middle volatility with a low mean, whereas the last one is a high mean state with minimal volatility. For example, in the high-voatility state, the means are negative and range from -0.283% for Equity Hedge to -0.160% for Macro/CTA. In the low-volatility state, the means are positive and range from 0.001% for Macro/CTA to 0.056% for Event Driven. In summary, the low-volatility state is typically paired with positive means, whereas the high-volatility state is paired with negative means. The high-volatility state is characterized by a crisis state. For all hedge fund strategies, the transition probability matrices exhibit a high persistence level to remain within the previous state, with a higher probability of remaining in the low-volatility state than in the middle-volatility state and in the middle-volatility state than in the high-volatility state.

Figure 1 visualizes the structure and evolution of the three-state Markov regime-switching model in more detail. A directed graph shows the Markov chain in which the states in the chain are depicted as nodes, and feasible transitions between states as directed edges. A feasible transition is a transition whose probability of occurrence is greater than zero. When the matrix entry p_{ij} is zero, the edge connecting states i and j are removed. The graph shows only a feasible transition between states. A self-loop indicates the transition

Table 2：Estimation results of the three-state regime-switching model

	Equity Hedge		Event Driven		Macro/CTA		Relative Value Arbitrage	
Three-state model								
Risk and return trade-off	σ	μ	σ	μ	σ	μ	σ	μ
High vol. state:	0.915	-0.283***	0.672***	-0.195***	0.871**	-0.160**	0.872**	-0.273***
Middle vol. state:	0.443***	-0.027	0.300***	0.001	0.358***	0.030***	0.234***	0.010
Low vol. state:	0.246***	0.053***	0.168***	0.056***	0.218***	0.001	0.106***	0.023***
Transition probability		Expected duration		Expected duration		Expected duration		Expected duration
High vol. sate:	0.921	12.7 days	0.936	15.6 days	0.894	9.4 days	0.884	8.6 days
Middle vol. state:	0.946	18.5 days	0.951	20.4 days	0.964	27.6 days	0.938	16.1 days
Low vol. state:	0.976	41.1 days	0.972	36.1 days	0.968	31.4 days	0.971	34.0 days

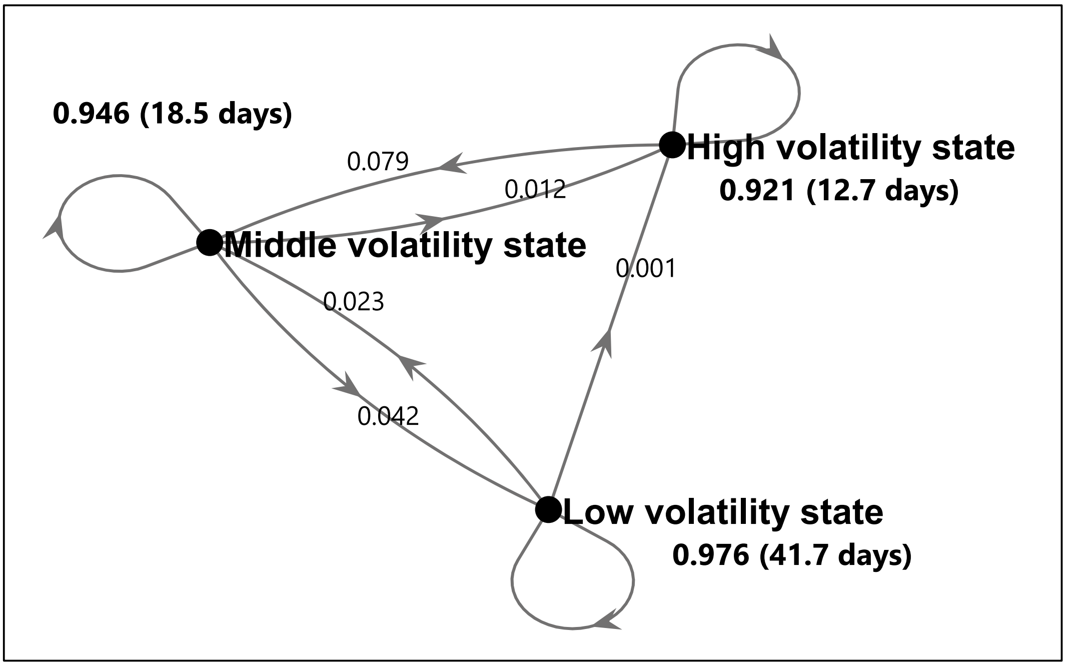
The asterisks refer to the level of significance, *= 10%, ** = 5%, and *** = 1%.

from state i back to itself, with nonzero probabilities P_{ii}. Self-transition probabilities imply that the values of p_{11}, p_{22}, and p_{33} indicate the probability of staying in the same state, which are known as the state inertia or persistence. The empirical probabilities of self-transition P_{ii} are denoted by circuits, whereas the expected durations are shown in parentheses. A walk between states i and j is a sequence of connected states that begins at i and ends at j.

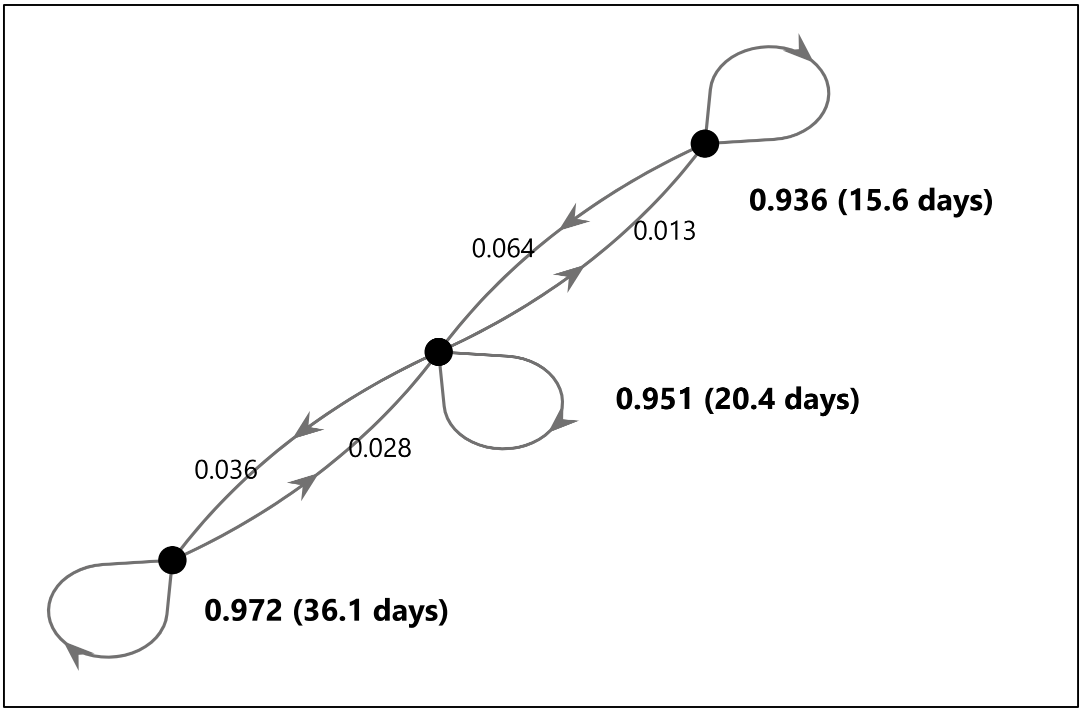
The Markov chains for all indices reveal that the high-volatility state is less likely than the middle volatility state, whereas the middle volatility state is less likely than the low-volatility state, although all regimes are likely to persist once they occur. The high values of the self-transition probabilities indicate a high persistence level that remains within the previous state. These high probabilities indicate the presence of persistent regimes, which play an important role in generating volatility clustering. For example, Equity Hedge exhibits that the self-transition probabilities of the high, middle, and low-volatility states are 0.921, 0.946, and 0.976, respectively, which suggests considerable state dependence. The corresponding expected durations in a regime are approximately 12.7, 18.5, and 41.1 days, respectively. The other three hedge fund indices indicate that the low-volatility state is typically paired with the longest expected duration, whereas the high-volatility state is paired with the shortest expected duration.

Figure 1 : Directed graph of the Markov Chain

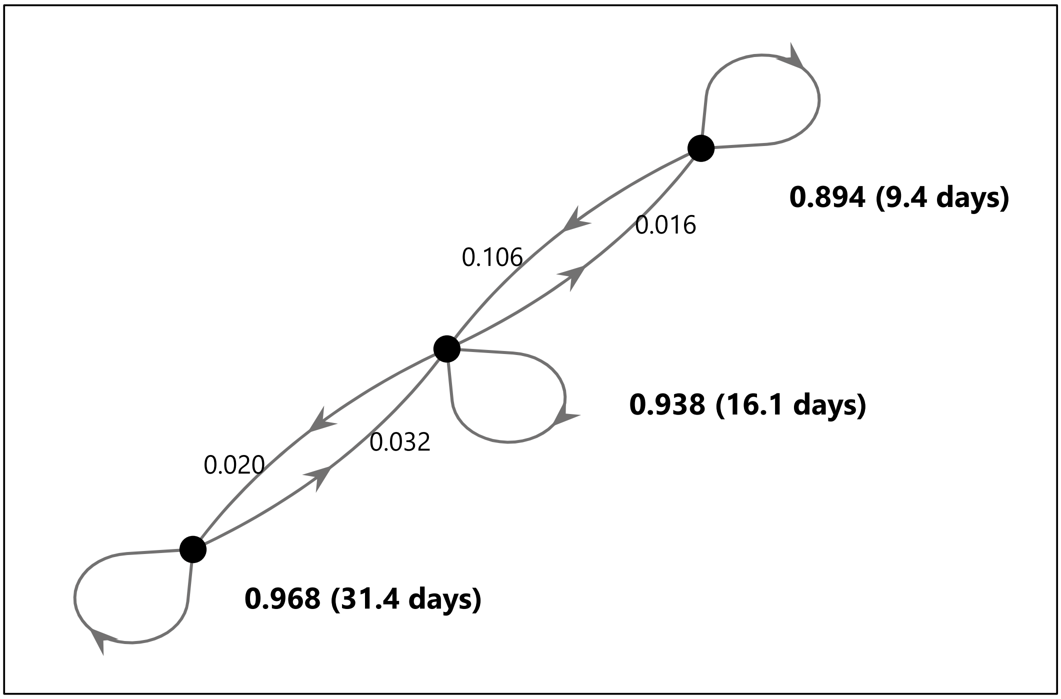
A：Equity Hedge



B：Event Driven



C：Macro/CTA



D：Relative Value Arbitrage

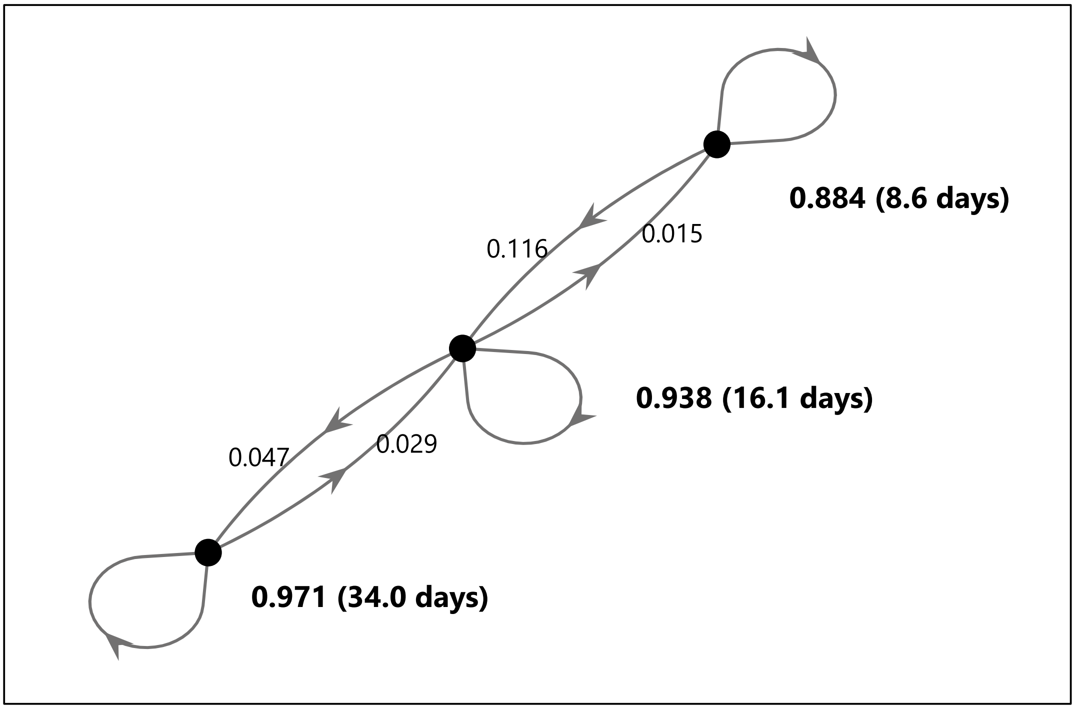


Figure 2 presents the distribution of each state in the three regimes among the four strategies. The high-volatility state provides an accurate description of the left-hand tail of the return distribution, which spans from 4.7% of Relative Value Arbitrage to 8.4% of Macro/CTA. This allows us to explain “infrequent” events in the data-generating process of hedge fund index returns, especially for the left-hand tail behavior during the financial crises.

We then concentrate explicitly on the high-volatility state. In Figures 3 plots the estimated smoothed probabilities of being in the high-volatility state of the four hedge fund index returns. The smoothed regime probability is constructed using a full sample of observations. These smoothed probabilities provide insights into the timing of the crisis state in each hedge fund strategy. Although a variety of the evolution of the high-volatility state exists among the four strategies, the high-volatility states of all strategies clearly jumped up during the global financial crisis of 2007 and 2009.

Figure 2：Distribution of the three states

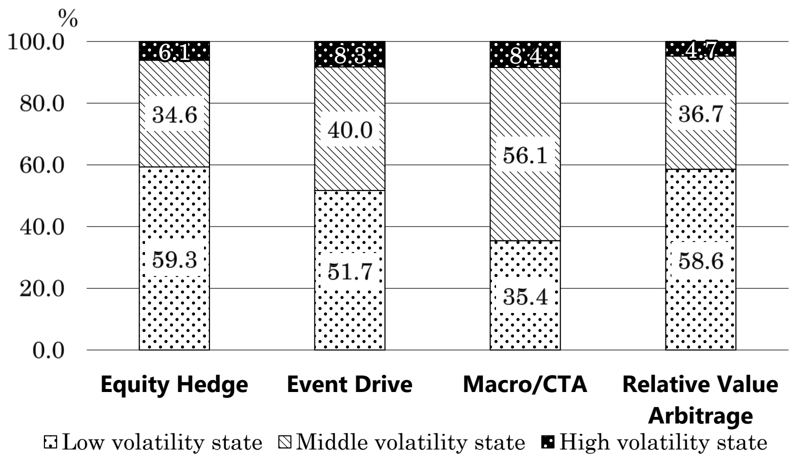
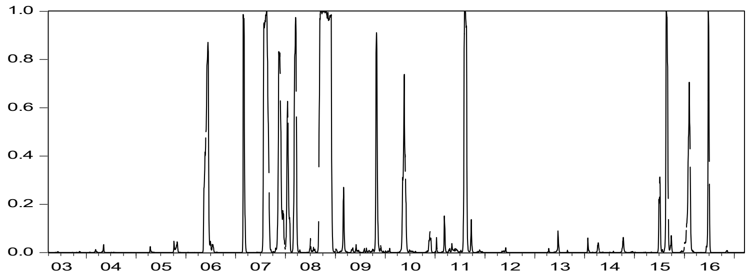
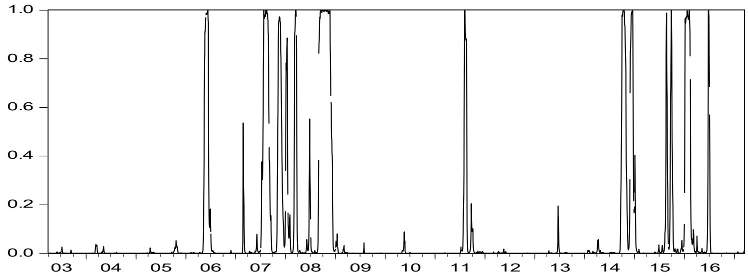


Figure 3：Smoothed regime probabilities of being in a high-volatility state

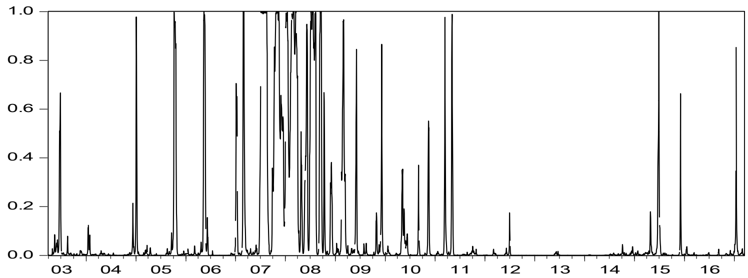
A：Equity Hedge



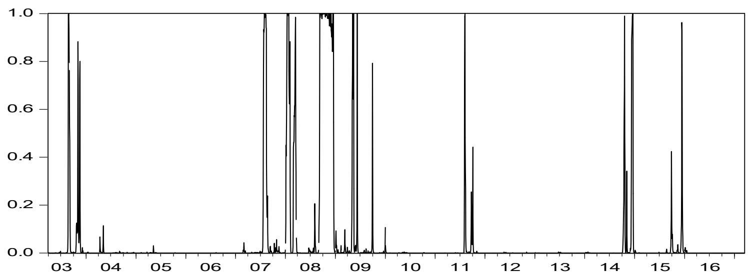
B：Event Driven



C：Macro/CTA



D：Relative Value Arbitrage



4. Hedge Fund Contagion

When all four indices are simultaneously in their high-volatility state, hedge fund contagion among the four strategies might have occurred, which provides an accurate, and sometimes fact-based representation of left-tail events. This commonality can be captured by the joint probability of being in the high-volatility state for all strategies, $J_{p,t}$. It is computed as the product of four probabilities of a high-volatility state in each strategy i, given the historical data up to and including data t.4)

$$J_{p,t} = \prod_{i=1}^{4} Prob\left(s_t^i = 1 \middle| r_t^i\right), \tag{8}$$

where $r_t^i \equiv (r_t^i, r_{t-1}^i, \ldots, r_1^i)$.

4.1 Short-lived contagion

Whether sharp contagion occurs among hedge fund strategies is identified by examining the coincidence of a high-volatility state across the four strategies. Figure 4 plots the estimates of the joint probability of the high-volatility state, in which this joint probability is close to zero for most of the sample period. However, the spikes (i.e., 90% and over) in the joint probabilities clearly occurred only during the financial crisis of 2007-2009: the Quants Meltdown, Bear Sterns collapse, and collapse of Lehman Brothers.

First, the joint probability jumps from approximately 0% on July 19, 2007, to 90.1% at 31, continued at high level, reached a peak of 98.2% on August 9, then subsided to 36.9% on August 15. The peak in the joint probability coincides with the Quants Meltdown of August 2007, triggering the US subprime crisis.5) At first the probability of a high-volatility state of Macro/CTA increased from 69% to 100% on July 5, 2007, and then those of the other three strategies followed Macro/CTA until July 26.

Second, the joint probability jumps up again from 9.1% on March 6, 2008, to 95.9% on March 17 and then dropped to 9.1% on March 20, the week of the Bear Sterns collapse (Sunday, March 16). In this case, initially the probability of the high-volatility state of Macro/CTA preceded the jump up to 95% on February 14, and those of the other three strategies increased to 90% on February 15, and then the joint probability reached its peak on Monday, March 17.

Third, the joint probability suddenly increased early in September and jumped to 85.6% on September 9 to almost 100% of the peak on September 15, continuing to over 99% until the 19th, the collapse of Lehman

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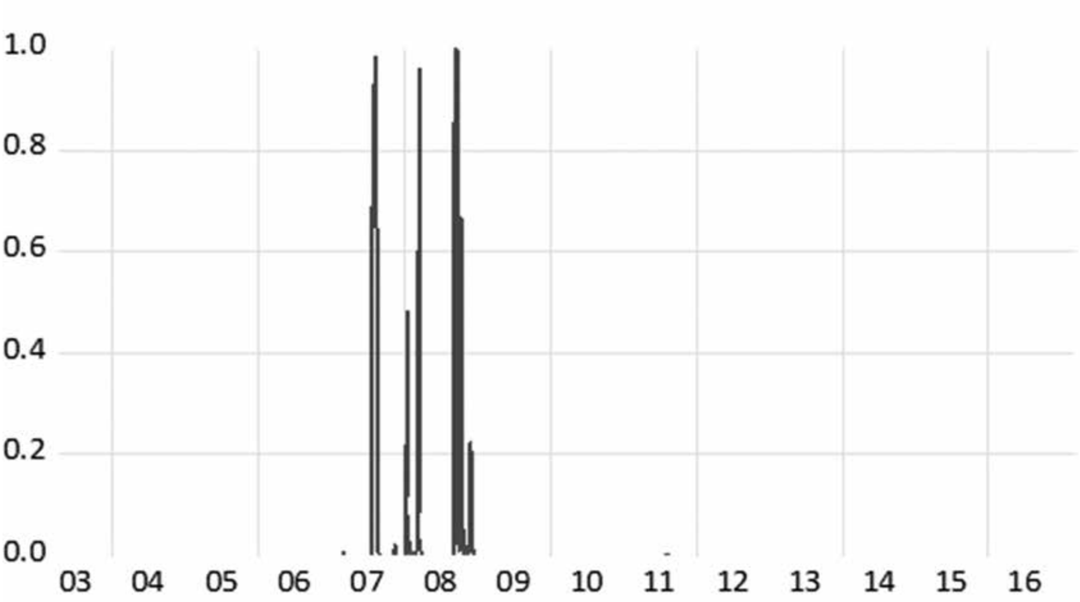
4) Billio et al. [2010] discuss the dependence when all four indices are simultaneously in their high volatility state and compute it based on their joint probabilities.

5) During the week of August 6, several prominent hedge funds experienced unprecedented losses. Model-driven long/short equity funds using quantitative strategies were severely affected on Tuesday (August 7) and Wednesday (August 8), and the S&P 500 lost nearly 3% on Thursday, August 9. (See Khandani and Lo [2007]).

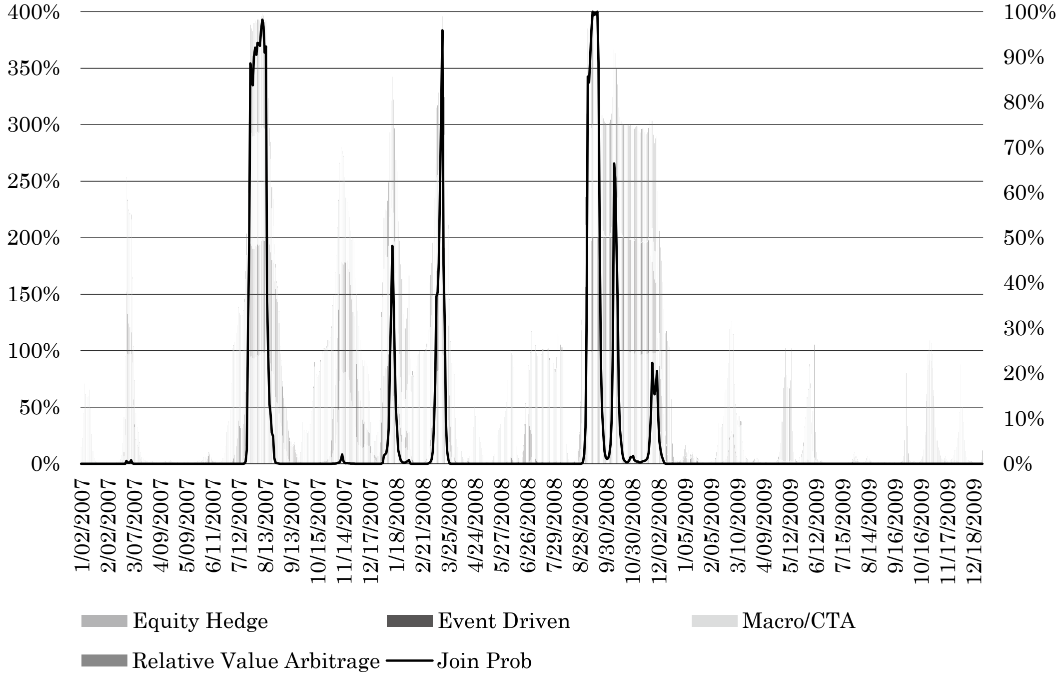
Figure 4：Joint probability of being in a high-volatility state



Brothers (Monday, September 15). During the financial crisis of 2007-2009, the periods over 90% in the joint probability of being in a high-volatility state among the four strategies are about two weeks from July 31 to August 14 for the Quants Meltdown of August 2007, only one day of March 17, 2008, for the Bear Sterns collapse, and nine days from September 11-19, 2008, for the collapse of Lehman Brothers. These facts indicate that hedge fund contagions occurred three times during the financial crisis of 2007-2009, but were quite short-lived events and were not persistent.

Figure 5 plots the daily summed probabilities of being in a high-volatility state across the four hedge fund indices from 2007 to 2009, in addition to the joint probabilities. The summed probabilities are considered as the aggregate level of distress in the hedge fund industry. We find that Macro/CTA played a large role at starting the short-lived hedge fund contagion. Macro/CTA moves initially to the high-volatility state, and the common condition for the appearance of hedge fund contagion is the joint occurrence of the high-volatility state between Macro/CTA and Relative Value Arbitrage. Macro/CTA is a directional strategy, and Relative Value Arbitrage is a mispricing strategy. Their primary investment theses present a striking contrast. Macro/ CTA strategy is predicted on predicted or future movements in the underlying instruments, while Relative Value Arbitrage strategy is predicted on realization of a primary discrepancy between related securities. HFR [2020] explains that investment managers of both strategies employ a variety of fundamental and quantitative techniques to establish their opposite investment thesis, while the two strategies often employ common techniques.

Figure 5：Summed and joint probability of being in a high-volatility state



4.2 Tail dependence structure

Tail dependence is the tendency toward larger correlations in the tails of the return distribution. Jondeau et al. [2007] point out that this property is time-varying cross-correlation as one of the stylized facts for asset returns. This implies that the correlation between asset returns tends to increase during high-volatility periods, particularly during a crisis. For example, Gentle [2020] reports that correlations of daily returns of the three indices (i.e., DIJA, S&P500, and Nasdaq) and the stock of Intel for the period of 2008 and 2009 are much higher than the correlations for the longer period of 1987 and 2017.

For hedge fund index returns, Table 3 reports the dependence structure among the daily returns of each strategy. Interestingly, the daily returns of the hedge fund indices exhibit quite different features of tail dependence. Correlations between Equity Hedge and Relative Value Arbitrage and those between Event Driven and Relative Value Arbitrage indeed tend to increase during the crisis period. Conversely, the correlation between Equity Hedge and Event Driven does not change. Strikingly, correlations between Macro/CTA and the other three strategies decrease and become even more negative during the crisis period. It is possible to say that the uniqueness of the tail dependence structure of Macro/CTA and other strategies may strongly affect the values of the joint probabilities of a high-volatility state.

Table 3：Tail dependence among the four hedge fund index daily returns

Daily returns	Full sample period: 2003/4/2-2017/3/16 & Crisis period: 2008/1/2-2009/12/31											
	EH			ED			Macro/CTA			RVA		
	Full		Crisis	Full		Crisis	Full		Crisis	Full		Crisis
EH		**1.00**		0.763	＞	0.756	0.264	＞	-0.055	0.419	＜	0.501
ED	0.763	＝	0.756		**1.000**		0.212	＞	-0.066	0.451	＜	0.548
Macro	0.264	＞	-0.055	0.212	＞	-0.066		**1.000**		0.114	＞	-0.023
RVA	0.419	＜	0.501	0.451	＜	0.548	0.114	＞	-0.023		**1.000**	

Table 4：Correlations of the probability of being in the high-volatility state among the hedge fund strategies

Daily returns	Full sample period: 2003/4/2-2017/3/16 & Crisis period: 2007/1/2-2008/12/31											
	EH			ED			Macro/CTA			RVA		
	Full		Crisis	Full		Crisis	Full		Crisis	Full		Crisis
EH		**1.000**										
ED	0.785	＜	0.915		**1.000**		0.320					
Macro	0.362	＞	0.210	0.320	＞	0.241		**1.000**				
RVA	0.612	＜	0.775	0.578	＜	0.711	0.275	＞	0.106		**1.000**	

Table 4 shows the correlations of the probability of being in a high-volatility state among hedge fund strategies. Similar to Table 3, the correlation structure of the probability of being the high-volatility state exhibits that correlations among Equity Hedge, Event Driven, and Relative Value Arbitrage tend to increase during the crisis period. However, correlations of Macro/CTA and the other three strategies decrease during the period of 2007-2008. The correlation of the probabilities of being in a high-volatility state between Macro/CTA and Relative Value Arbitrage is key to the emergence of hedge fund contagion.

5. Conclusion

In this study, we investigated the volatility behavior of the daily returns of hedge fund strategy indices, especially focusing on the inter-strategy contagion in left-hand tail events by using a Markov regime switching model. In the context of the regime-switching approach, using daily data represents an important innovation because earlier research on hedge fund contagion mainly utilized only monthly data. The main benefit of using daily data is that short-lived hedge fund contagions during the financial crisis of 2007-2009 can be detected and insights into each timing and duration can be provided. We found strong evidence of the switching behavior in hedge fund index returns, and that the short-lived hedge fund contagion occurred three times during the financial crisis of 2007-2009. These contagions were linked to specific crisis episodes: Quants Meltdown of August 2007, Bear Sterns collapsed in March 2008, and collapse of Lehman Brothers in September 2008. On these occasions, Macro/CTA plays a significant role in emerging hedge fund contagion and is short-lived. Given that hedge fund contagion is captured by the coincidence of being in a high-volatility state among hedge fund indices, tail dependence, that is, the correlation structures of the probability of being in a high-volatility state, should be examined. The correlations among Equity Hedge, Event Driven, and Relative Value Arbitrage tend to increase during the crisis period. In contrast, the correlations of Macro/CTA with the other three strategies decreased during the crisis period. Thus, Macro/CTA may offer effective protection against systemic risk by shortening the duration of the inter-strategy hedge fund contagion.

Appendix：HFRX Global Hedge Fund Index

Single Strategy	Description
Equity Hedge	Equity Hedge strategies are the equity-based strategy, known as long/short equity, typically maintaining at least 50%, and, in some cases, substantially entirely invested in equities in both long and short positions and equity derivative securities. The investment decision includes both quantitative and fundamental technique strategies that can be broadly diversified or narrowly focused on specific sectors, and frequently employed leverage.
Event Driven	A strategy that specifically focuses on corporations involved in special situations or significant restructuring events, such as mergers, liquidations, and insolvencies. The goal is to take advantage of price anomalies triggered by special events. Securities include a variety of types in the capital structure from most serious to most junior or subordinated and frequently involved additional derivative securities. Investment theses are typically predicted on fundamental characteristics (as opposed to quantitative), with the realization of the thesis predicated on a specific development exogenous to the existing capital structure.
Macro/CTA	Macro/CTA strategies are directional strategies based on the prediction to future macroeconomic movements, whose managers employ various techniques, both discretionary and systematic analysis, quantitative and fundamental approaches. Although some strategies employ relative value techniques, the primary investment thesis of Macro/CTA is predicated in future movements in underlying instruments, rather than realizing a valuation discrepancy between securities in Event Driven. In a similar manner, both Macro/CTA and Equity Hedge managers may hold equity securities, and the overriding investment thesis is predicated on the impact movements in underlying macroeconomic variables that may have on security prices. This is opposed to Equity Hedge in which the fundamental characteristics of a company are the most significant and integral to investment thesis.
Relative Value Arbitrage	Relative value arbitrage strategies are the strategies that attempt to take advantage of temporarily mispricing valuations in the relationship between multiple securities. Managers employ various fundamental and quantitative techniques to establish investment theses. The security type involves a broad range across equity, fixed income, derivative, or other security types. Meanwhile, Relative Value Arbitrage is a non-directional strategy. The relative value arbitrage position may be involved in corporate transactions. However, as opposed to Event Driven exposures, investment thesis is predicated on realization of a pricing discrepancy between related securities as opposed to the outcome of corporate transactions.

Source: Hedge Fund Research [2020]

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